VARIABILITY IN SYNTHETIC EARTHQUAKE GROUND MOTIONS CAUSED BY SOURCE VARIABILITY AND ERRORS IN WAVE PROPAGATION MODELS

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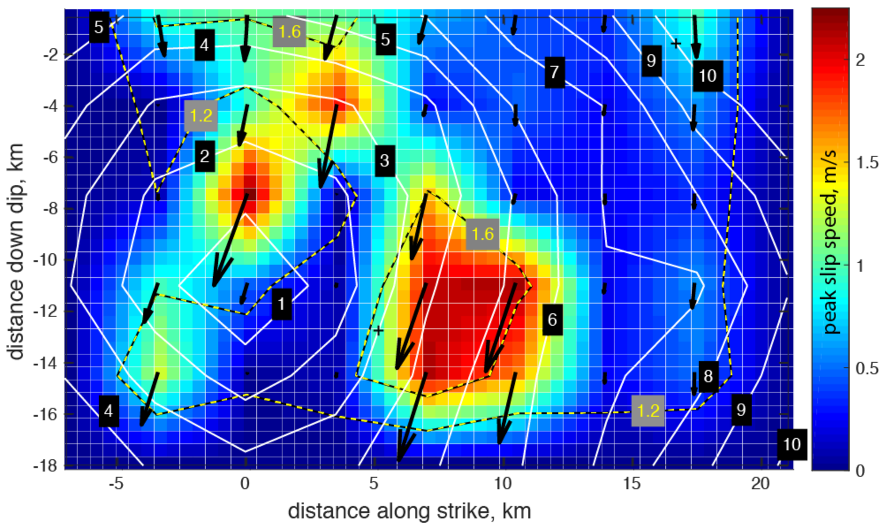
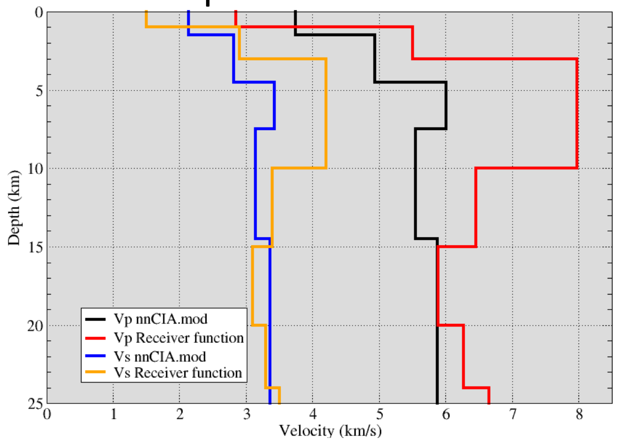
Waves and Ruptures

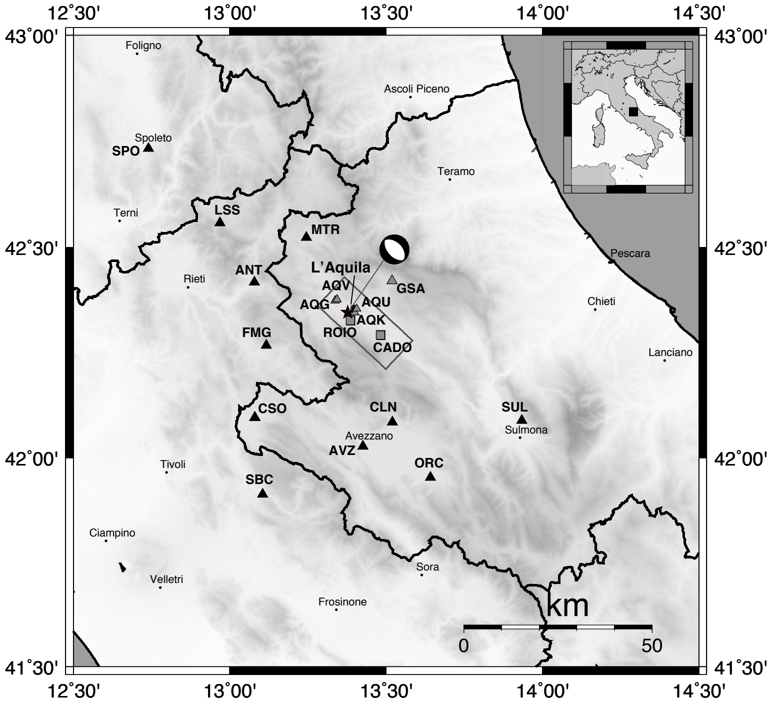
Given a kinematic slip model and a local seismic velocity structure, it is straightforward to calculate the resulting ground motions deterministically.

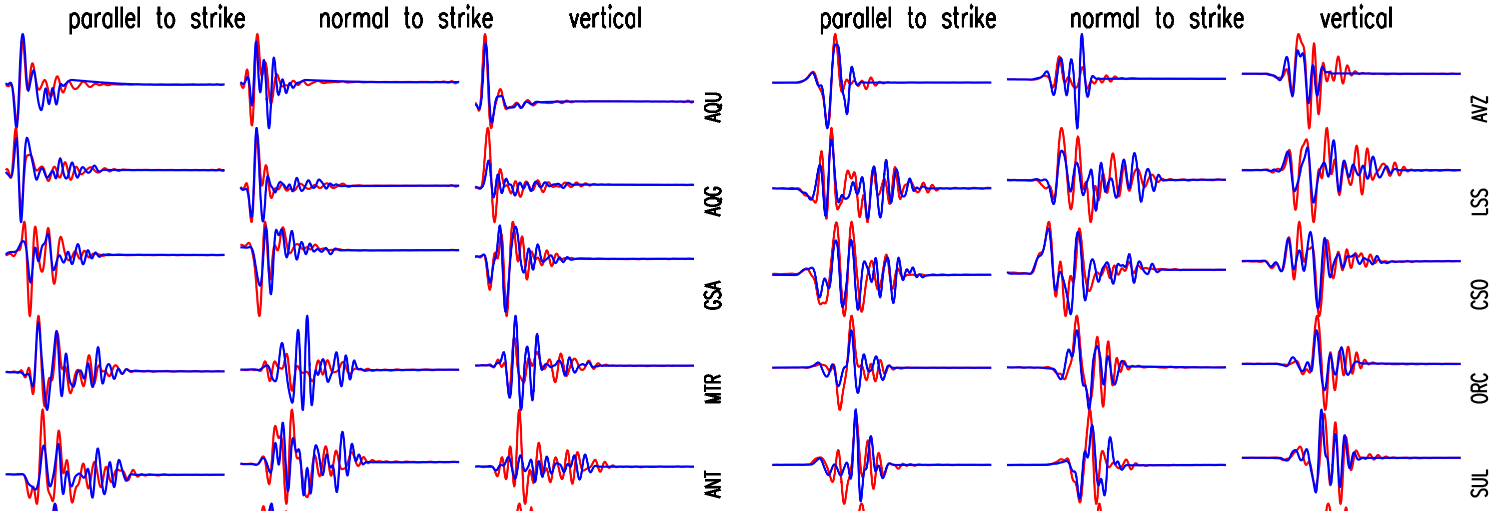
Best fitting L'Aquila rupture model



Vp & Vs 1D model







It is necessary to assume some Earth structure that is necessarily inaccurate

LITERATURE –

- ✓ Recently Yagi and Fukahata (2011) presented a method to include uncertainties in Green functions (*theory errors*) into an inversion for earthquake rupture behavior, by using a time-domain approach. They were possibly the first investigators to try to quantify the variation of the ground motions caused by errors in the Green's function.

- ✓ *Theory errors* are also included in Bayesian inversion (Duputel et al (2012), Minson et al (2014), Ragon et al. (2018)) through theoretical considerations.

None of the investigators actually measured Green's function errors.

OUTLINES -

1) Present an equation of ground velocity that includes the **Green function errors** (frequency domain);

2) Derive the expected variance γ^2 caused by Green function errors for a large earthquake;

3) Compute the **Green function errors** for a test case (L'Aquila region);

4) Compare these errors with the misfit of the best model for the 2009 L'Aquila event, Mw 6.1;

5) General discussion on source variabilities and green functions errors;

6) Future applications.

Details are written in the recent paper by Spudich et al. 2019

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Article Contents

Summary

Notes

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Variability in synthetic earthquake ground motions caused by source variability and errors in wave propagation models

FREE

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Summary

Numerical simulations of earthquake ground motions are used both to anticipate the effects of hypothetical earthquakes by forward simulation and to infer the behavior of the real earthquake source ruptures by inversion of recorded ground motions. In either application it is necessary to assume some Earth structure that is necessarily inaccurate and to use a computational method that is also inaccurate for simulating the wave field Green's functions. We refer to these two sources of error as ‘propagation inaccuracies,’ which

Green function errors

A major problem is that we do not know the seismic velocity structure perfectly and our methods for calculating traction Green's functions are inaccurate.

Let the true tractions on a fault at point caused by a point force in the j-direction at the observer at y be $\mathbf{g}^{(j)}(\mathbf{x},\omega;\mathbf{y})$

Let its numerical approximation based on an inaccurate velocity structure be $\tilde{\mathbf{g}}^{(j)}(\mathbf{x},\omega;\mathbf{y})$

Let its error be $\delta\mathbf{g}^{(j)}(\mathbf{x},\omega;\mathbf{y})$

so we have…

$$\mathbf{g}^{(j)}(\mathbf{x},\omega;\mathbf{y}) = \tilde{\mathbf{g}}^{(j)}(\mathbf{x},\omega;\mathbf{y}) + \delta\mathbf{g}^{(j)}(\mathbf{x},\omega;\mathbf{y})$$

Similarly, the relation between the true slip velocity, the assumed and the variation in slip velocity can be written:

$$\mathbf{s}(\mathbf{x},\omega) = \tilde{\mathbf{s}}(\mathbf{x},\omega) + \delta\mathbf{s}(\mathbf{x},\omega)$$

Ground velocity in frequency domain:

$$d_j(\omega,\mathbf{y}) = v_j(\omega,\mathbf{y}) + n_j(\omega,\mathbf{y}).$$

$v_j(\omega,\mathbf{y})$ is the Fourier Transform of the j component of ground velocity at location y

$\mathbf{s}(\mathbf{x},\omega)$ is the Fourier Transform of the slip velocity vector at location x

Noise-free Ground velocity:

$$v_j(\omega,\mathbf{y}) = \int\limits_{\mathbf{x}\in A} \mathbf{s}(\mathbf{x},\omega)\bullet \mathbf{g}^{(j)}(\mathbf{x},\omega;\mathbf{y})\,dA$$

$$= \int\limits_{\mathbf{x}\in A} \tilde{\mathbf{s}}\bullet\tilde{\mathbf{g}}^{(j)}dA + \int\limits_{\mathbf{x}\in A} \delta\mathbf{s}\bullet\tilde{\mathbf{g}}^{(j)}dA + \int\limits_{\mathbf{x}\in A} \tilde{\mathbf{s}}\bullet\delta\mathbf{g}^{(j)}dA + \int\limits_{\mathbf{x}\in A} \delta\mathbf{s}\bullet\delta\mathbf{g}^{(j)}dA.$$

Ground velocity in frequency domain

$$v_j(\omega,\mathbf{y}) = \int_{\mathbf{x}\in A} \tilde{\mathbf{s}} \bullet \tilde{\mathbf{g}}^{(j)} dA + \int_{\mathbf{x}\in A} \delta\mathbf{s} \bullet \tilde{\mathbf{g}}^{(j)} dA + \int_{\mathbf{x}\in A} \tilde{\mathbf{s}} \bullet \delta\mathbf{g}^{(j)} dA + \int_{\mathbf{x}\in A} \delta\mathbf{s} \bullet \delta\mathbf{g}^{(j)} dA.$$

δv_j^s is a measure of the aleatory variability in the ground velocity caused by rupture source variability.

δv_j^g is a measure of the epistemic variability in the ground velocity because errors in the geologic structure.

δv_j^{sg} is the ground velocity caused by the interaction of δg and δs. it might not be negligible depending on the amplitudes of δs and dδ.

These terms show how variations in the rupture model and errors in the Green's functions contribute to the total motion

Green function errors

The variation in ground velocity caused by errors in our Green's function is

$$\boxed{\delta v_j^g = \int_{\mathbf{x}\in A} \mathbf{s} \bullet \delta \mathbf{g}^{(j)} dA}$$

We compute the variance of γ^2 that can be related to the statistics of the Green's function error. This variance is a function of frequency, component and observation location.

Developing a frequency-domain equivalent to Yaki and Fukahata (2011) we discovered the following simple relation for the variance of the ground motion:

$$\boxed{\gamma_j^2(\omega) = \int\limits_{\mathbf{x}\in A} \int\limits_{\mathbf{x}'\in A} \tilde{s}_1^*(\mathbf{x},\omega)\, C_{11}^j(\mathbf{x},\mathbf{x}',\omega)\, \tilde{s}_1(\mathbf{x}',\omega)\, d\mathbf{x}\, d\mathbf{x}'}$$

$\gamma_j^2(\omega)$ is the variance in ground velocity of the j-th channel (single component of motion at a particular observation location)

$s_1^*(\mathbf{x},\omega)$ is the complex conjugate of the slip velocity in the dominant slip direction (called the '1' direction) at point x on the fault

$C_{11}^j(\mathbf{x}_1,\mathbf{x}_2,\omega) = E\left[\delta g_1^{j*}(\mathbf{x}_1)\delta g_1^j(\mathbf{x}_2)\right]$ is the covariance of the errors in the traction Greens functions

The equation has been obtained assuming a dominant component in the rupture

Derived covariance function

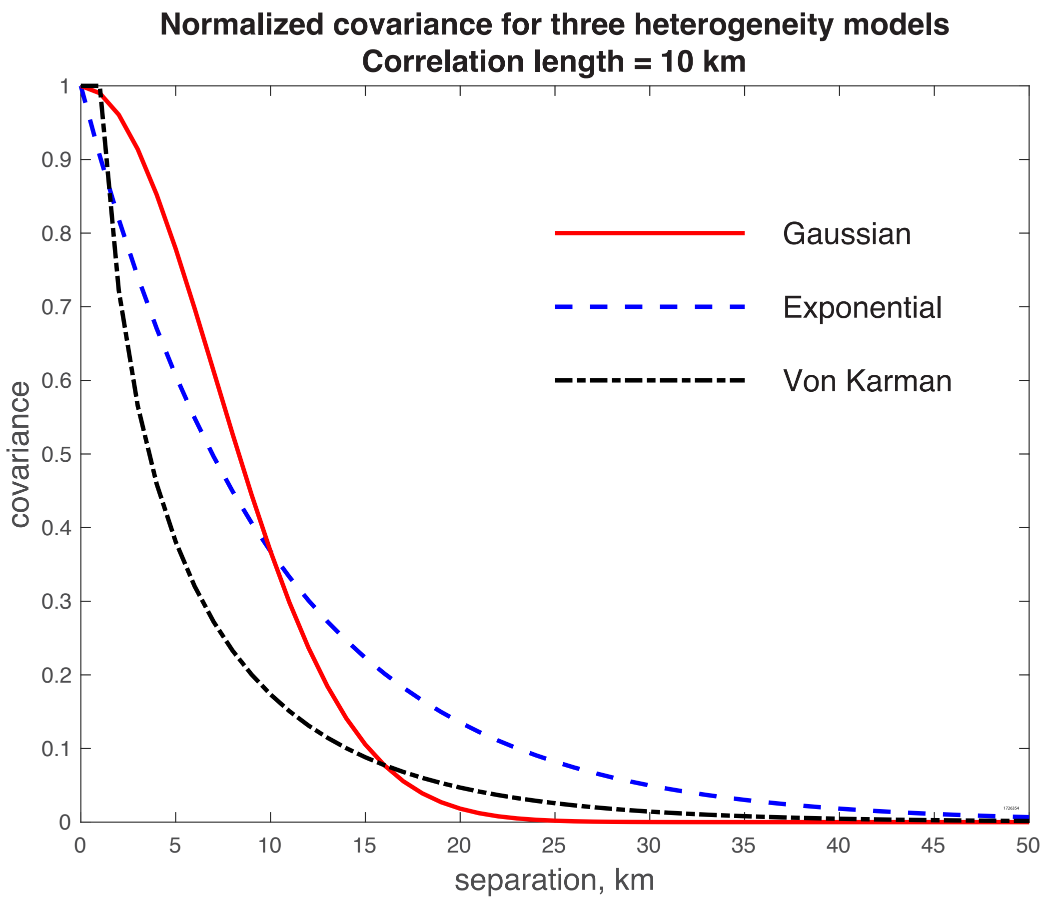
$$C_{11}^{j}\left(\mathbf{x}_1,\mathbf{x}_2,\omega\right)=E\left[\delta g_1^{j*}\left(\mathbf{x}_1\right)\delta g_1^{j}\left(\mathbf{x}_2\right)\right]$$

C^{J}_{11} is the covariance of the errors in the traction Greens functions at location x_1 on the rupture area A with the errors in the traction Green's functions at location x_2 on the rupture area for the j-th data channel (single component of motion at a particular observation location).

This covariance function allows us to make realistic estimates of the variance based on observed data quantifying Green's function error.

Covariance of green function errors

A simple model for this covariance function might be a function of the separation between points x1 and x2 on the rupture surface, with or without some dependence on frequency. This is one way of quantifying the spatial heterogeneity.



$$\boxed{\mathbf{C}^{j}\left(\mathbf{x}_1,\mathbf{x}_2,\omega\right)\propto f\left(\left|\mathbf{x}_1-\mathbf{x}_2\right|,\omega\right)}$$

Covariance of green function errors

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 91, NO. B6, PAGES 6465-6489, MAY 10, 1986

Finite Difference Simulations of Seismic Scattering; Implications for the Propagation of Short-Period Seismic Waves in the Crust and Models of Crustal Heterogeneity

ARTHUR FRANKEL
ROBERT W. CLAYTON

They used a 2D finite difference algorithm to model wave propagation in random heterogeneous media with three different autocovariance functions, a self-similar, an exponential, and a gaussian.

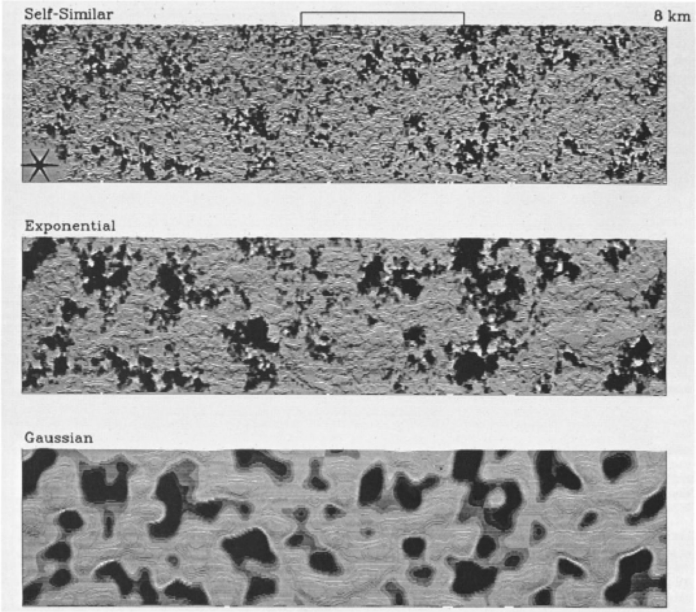


Fig. 1. Representations of the three types of random media considered in this study. Each medium shown has a correlation distance of 200 m. The amplitude of each horizontal line denotes the random component of the P or S wave velocity (V_P/V_S constant). Areas with higher than average velocity are shaded. In the top panel, the positions of the explosion source (star) and receiver array (bar) along the free surface are shown. This configuration was used in the simulations described in the section on high-frequency coda amplitude and cross correlation.

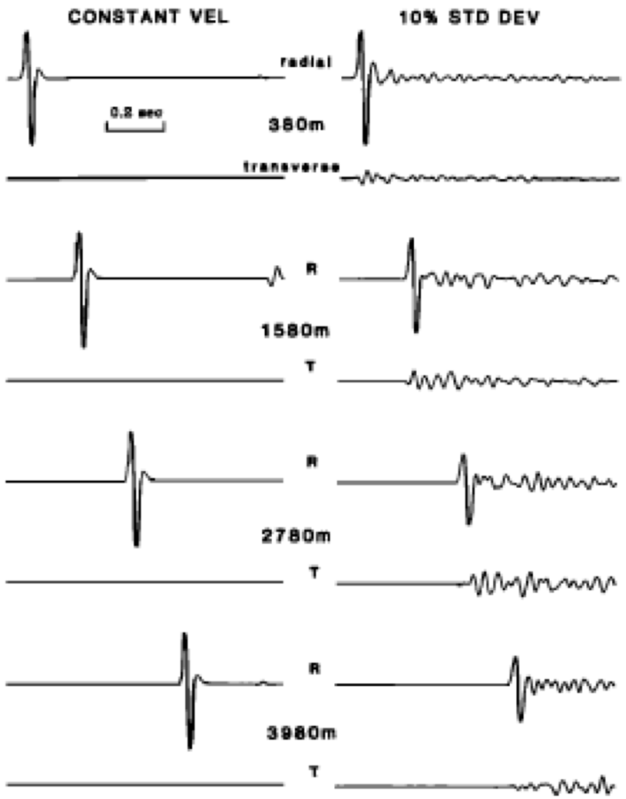


Fig. 4. (Left) Synthetic seismograms for a P wave propagating through a constant velocity medium (see Figure 2 for geometry). Radial and transverse components of velocity are shown for receivers from 380 to 3980 m from the source. Amplitudes of all synthetics shown in this paper are corrected for geometrical spreading. (Right) Synthetic seismograms for a P wave traveling through an exponential random medium ($\sigma_v = 10\%$, $a = 40$ m, $ka = 1.16$ at 30 Hz).

Frankel & Clayton (1986) specified the covariance of their random seismic velocity structures, and their variations in wave amplitude were the result of the random structures. We, on the other hand, are using observations of aftershock seismograms to look directly at the random variation of the traction *wave field.*

Covariance of green function errors

$$\gamma_j^2(\omega) = \int\limits_{\mathbf{x}\in A} \int\limits_{\mathbf{x}'\in A} \tilde{s}_1^*(\mathbf{x},\omega)\, C_{11}^j(\mathbf{x},\mathbf{x}',\omega)\, \tilde{s}_1(\mathbf{x}',\omega)\, d\mathbf{x}\, d\mathbf{x}'$$

To use our equation of the variance γ^2 , you must be able to estimate an accurate spatial covariance function.

$$C(\mathbf{x}_1,\mathbf{x}_2,\omega) \propto f(|\mathbf{x}_1 - \mathbf{x}_2|,\omega)$$

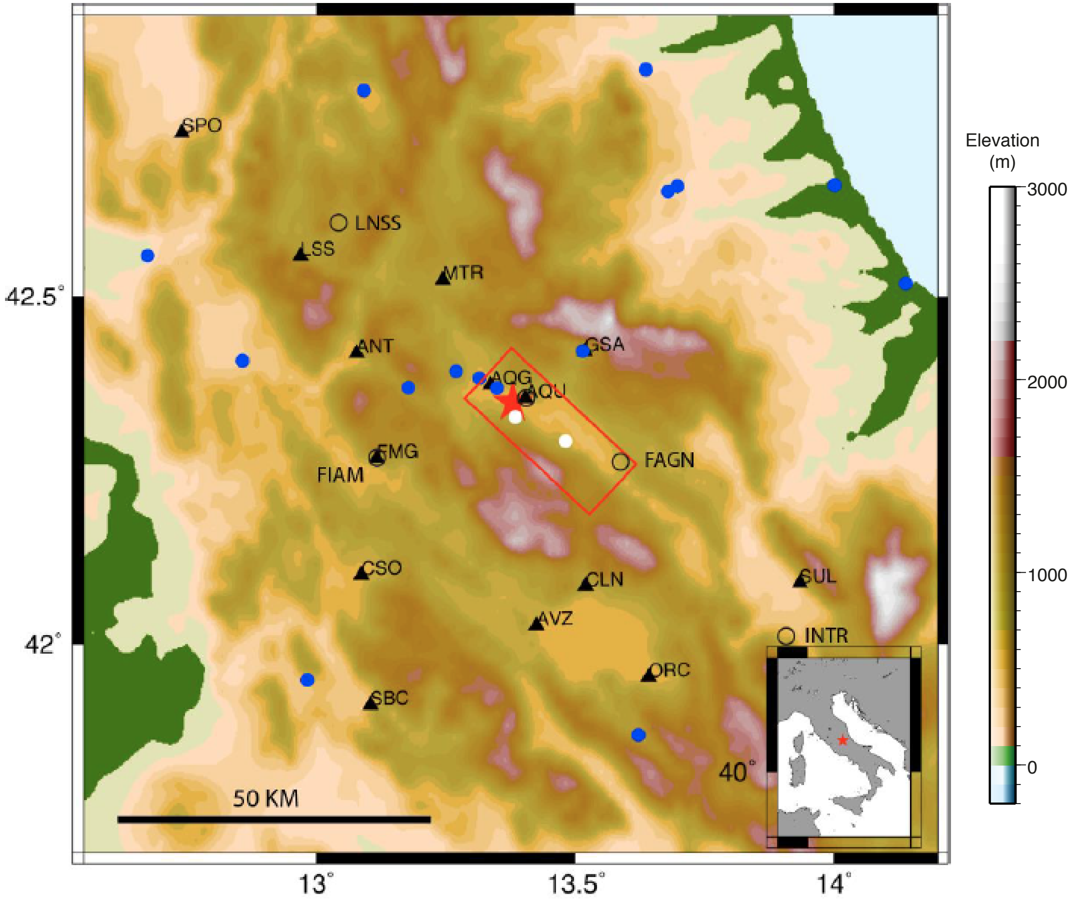
Two possibilities:

1) If you have recordings of many small earthquakes on the rupture surface of interest, you can treat them as empirical Green's functions and use them to calculate the error in your theoretical Green's functions.

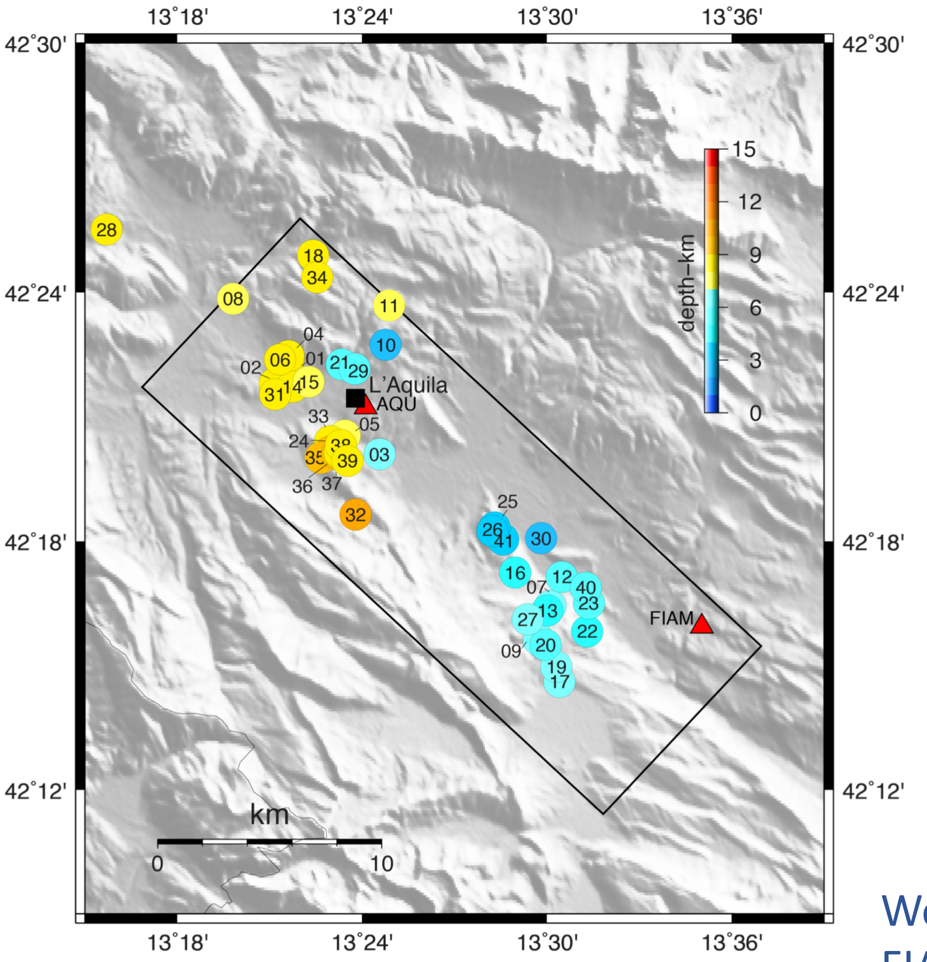
2) If you do not have recordings of many empirical Green's functions, it might be possible to infer the needed spatial covariances, following Frankel and Clayton (JGR, 1986), from coda-Q and teleseismic travel-time and amplitude anomalies.

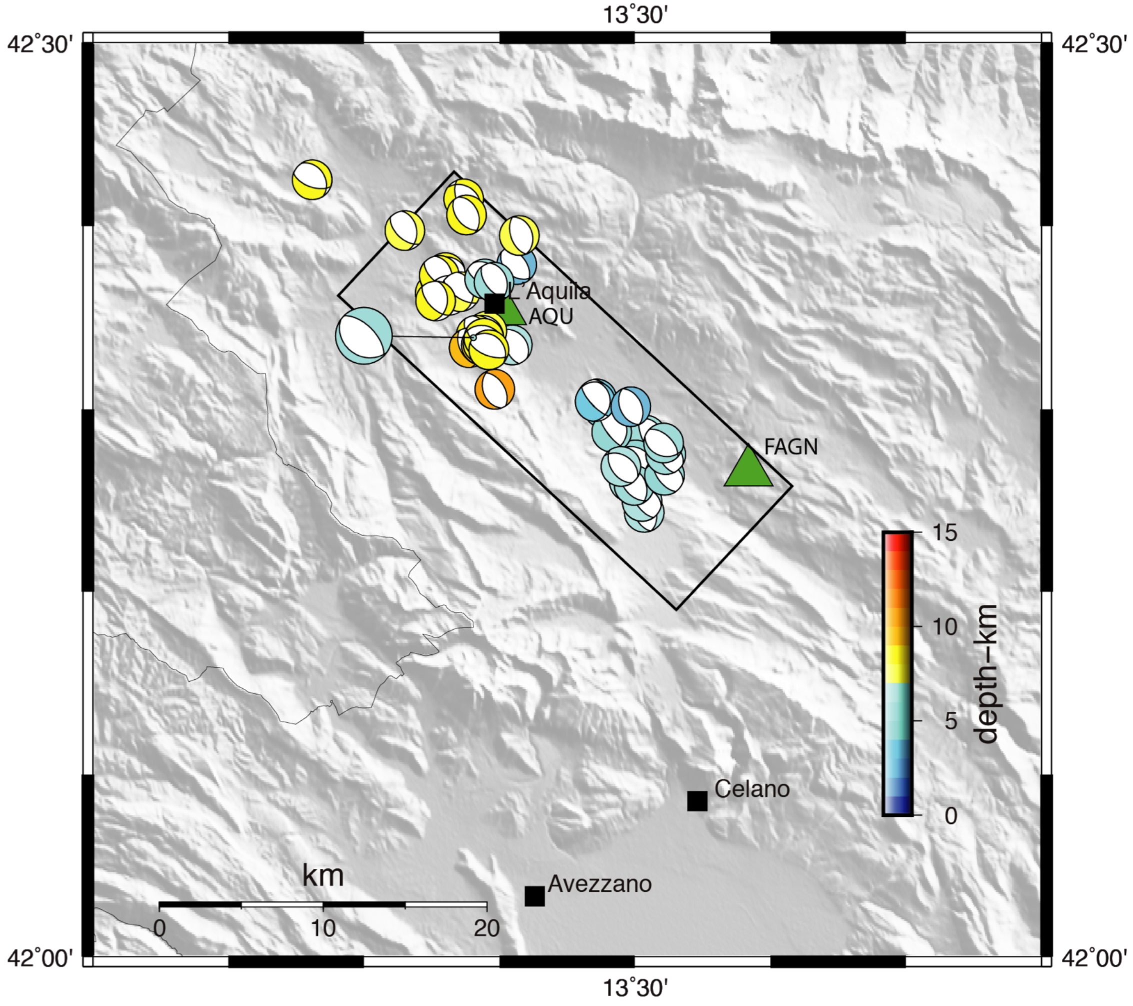
We have selected for study the 6 April 2009 M6.1 L'Aquila, central Italy, earthquake and its **on-fault** aftershocks, from which we derive covariance functions

Map of strong motion and other stations



Selected aftershock locations and mechanisms

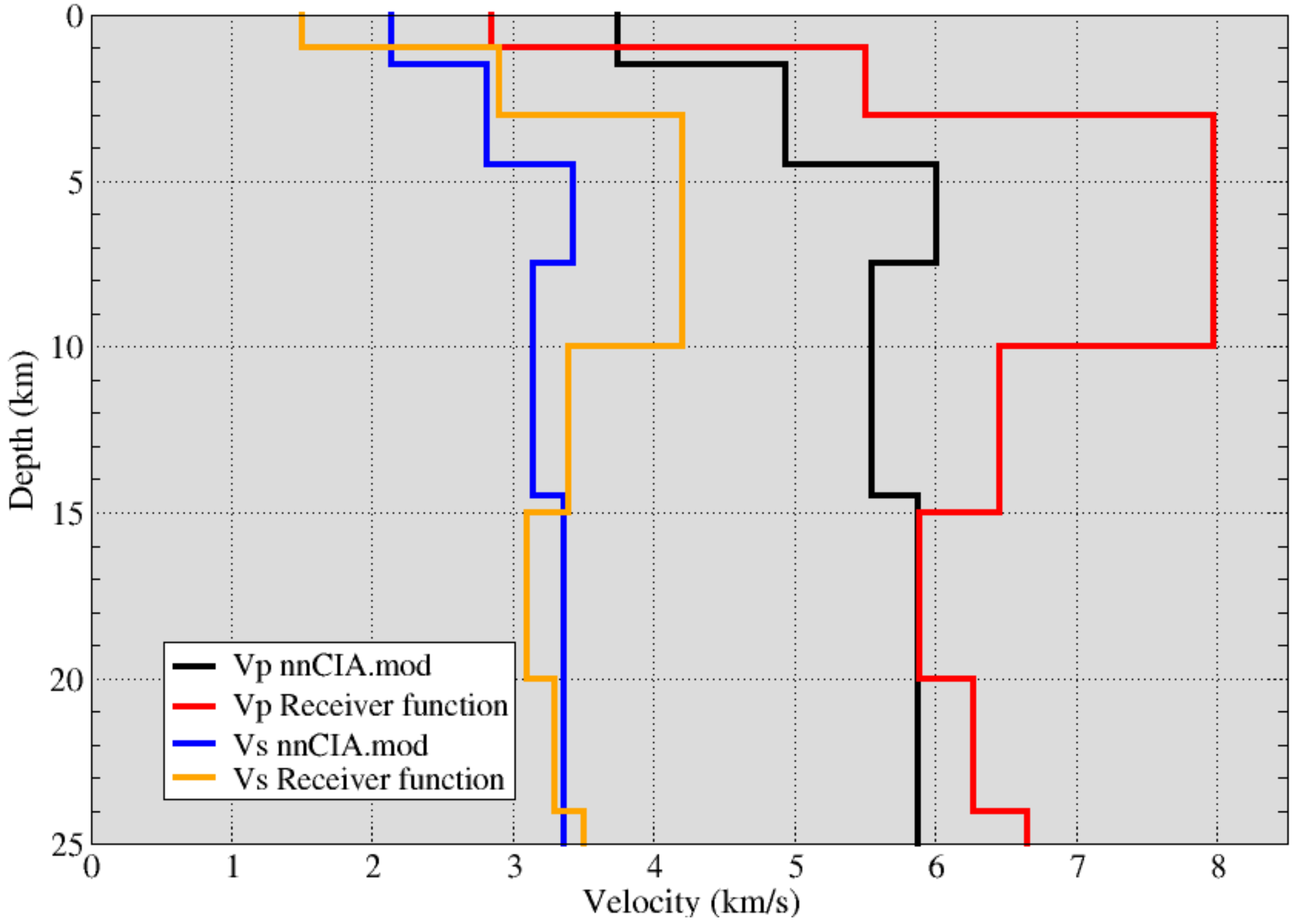




We chose as empirical Green's functions 37 events recorded by AQU and FIAM seismic stations which have Mw between 2.3 and 3.8, high signal to noise ratio, and focal planes within 30 degrees of the main shock mechanism

Covariance of green function errors – test case L'Aquila

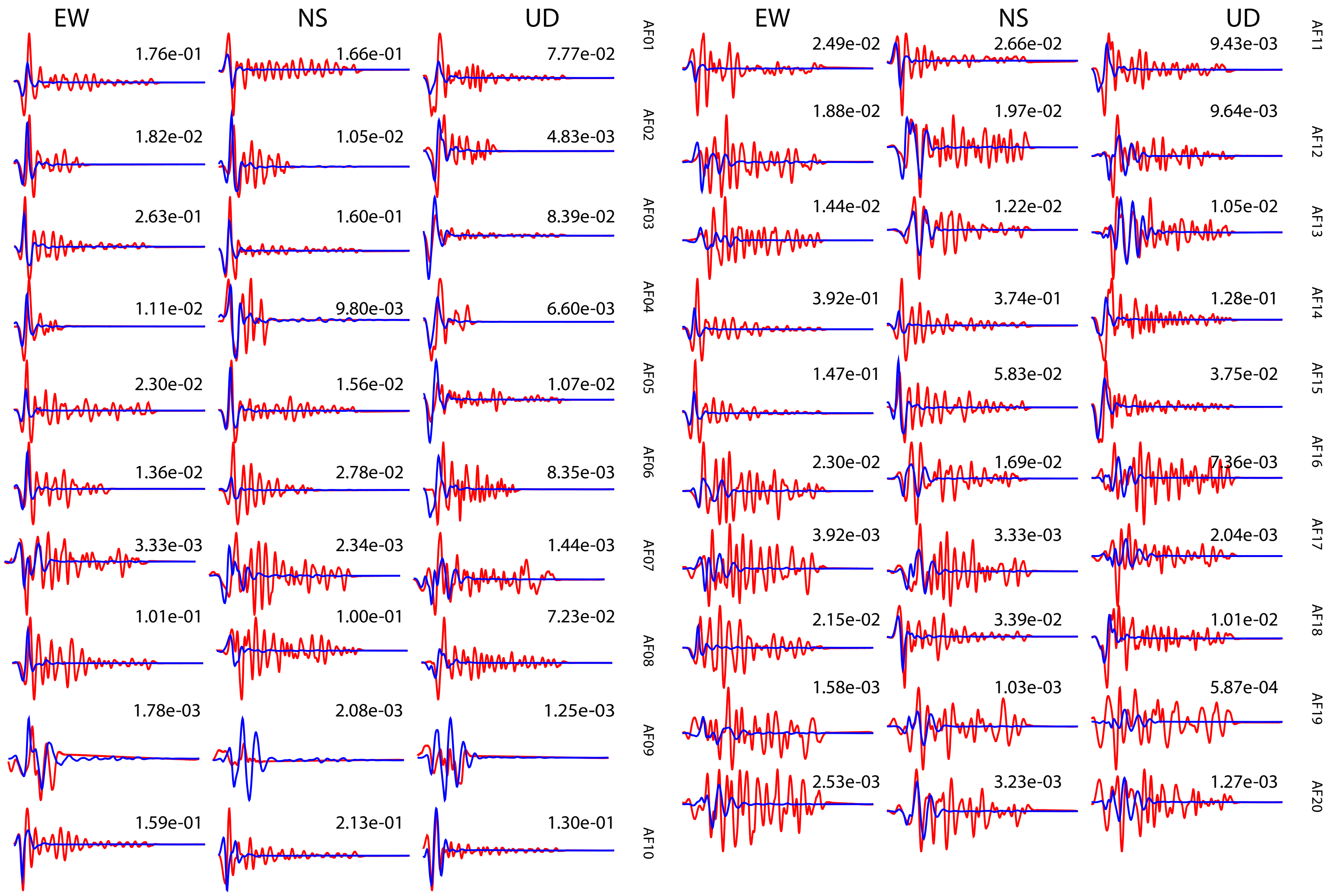
Two different seismic velocity structures, **CIA model**, shown by the black and blue curves, and the **receiver function (RF) model**, shown by the red and orange curves, were used to calculate point source synthetics at AQU and FIAM for these aftershocks.



Covariance of green function errors – test case L’Aquila

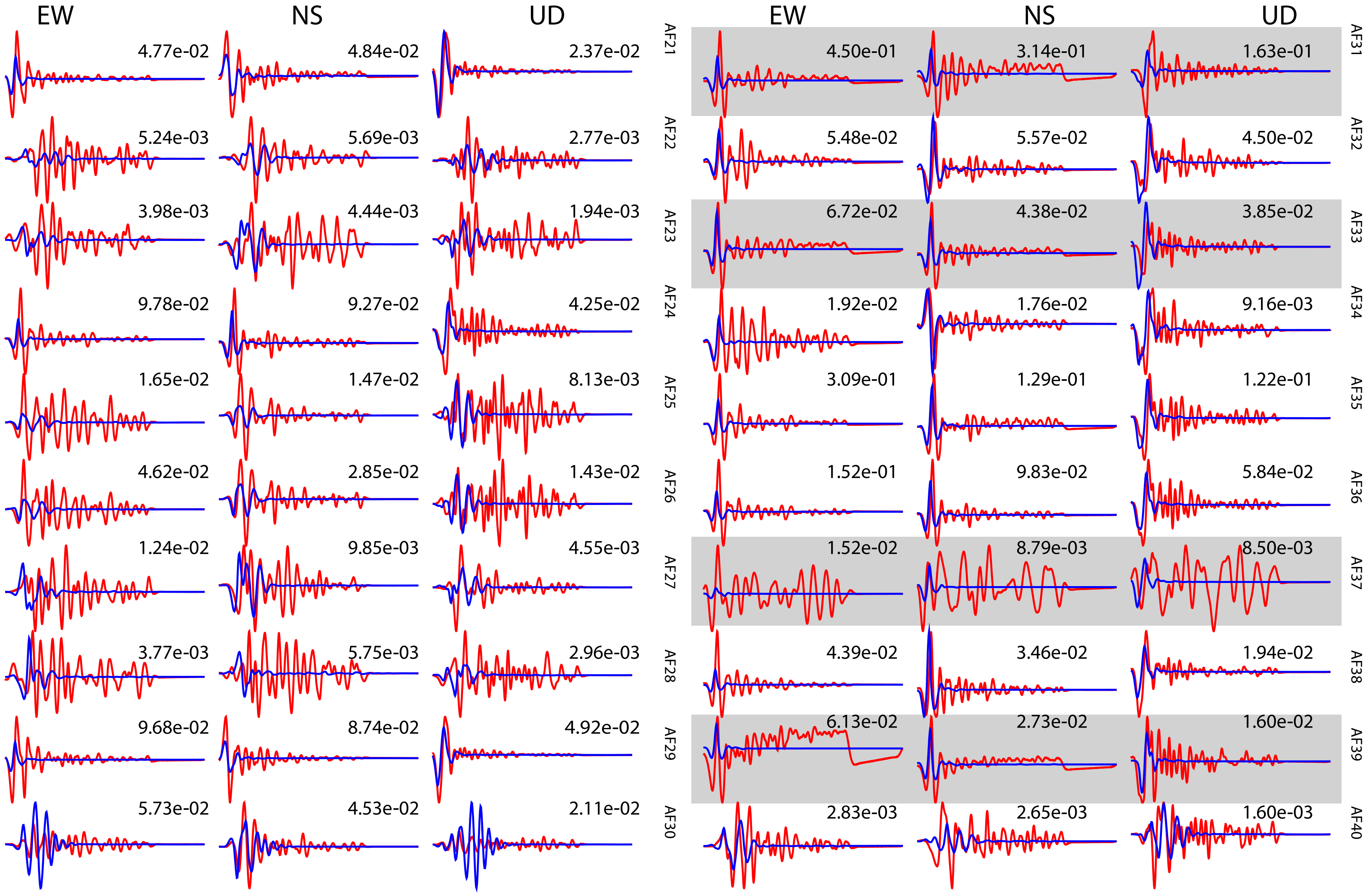
Forward models for point sources whose moment tensor solutions have been inferred from farther broadband stations at lower frequency band.

Observed ground velocity at **AQU** (red) and synthetic velocity (blue) for the RF structure for frequency band 0.02-0.5 Hz. Number is peak velocity of data seismogram. First 40s of total 60s seismograms are shown.



Covariance of green function errors – test case L'Aquila

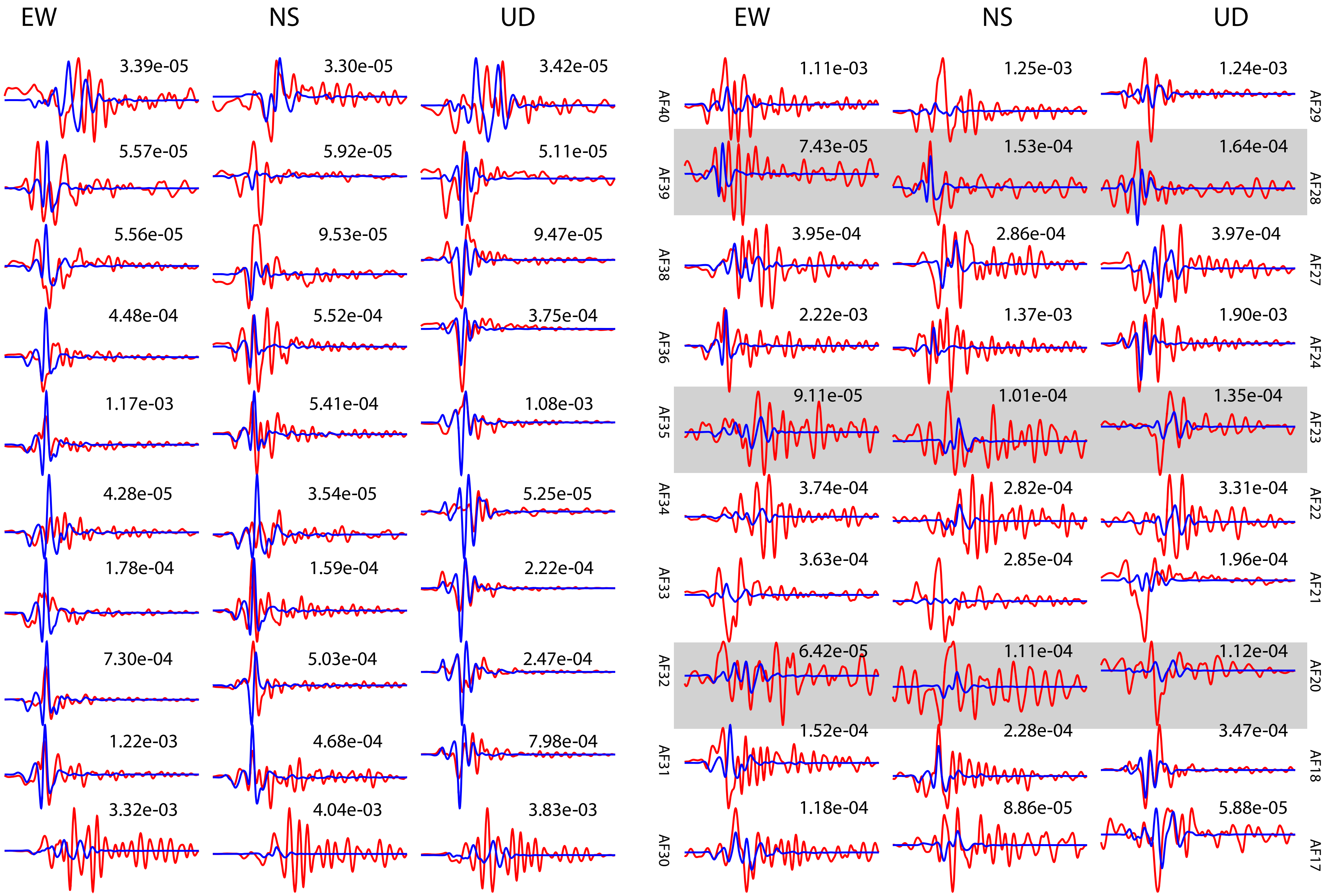
Observed ground velocity at **AQU** (red) and synthetic velocity (blue) for the RF structure, plotted at the same scale as the observations.



We removed from the analysis those data for which the observed data had obvious ground noise or processing glitches

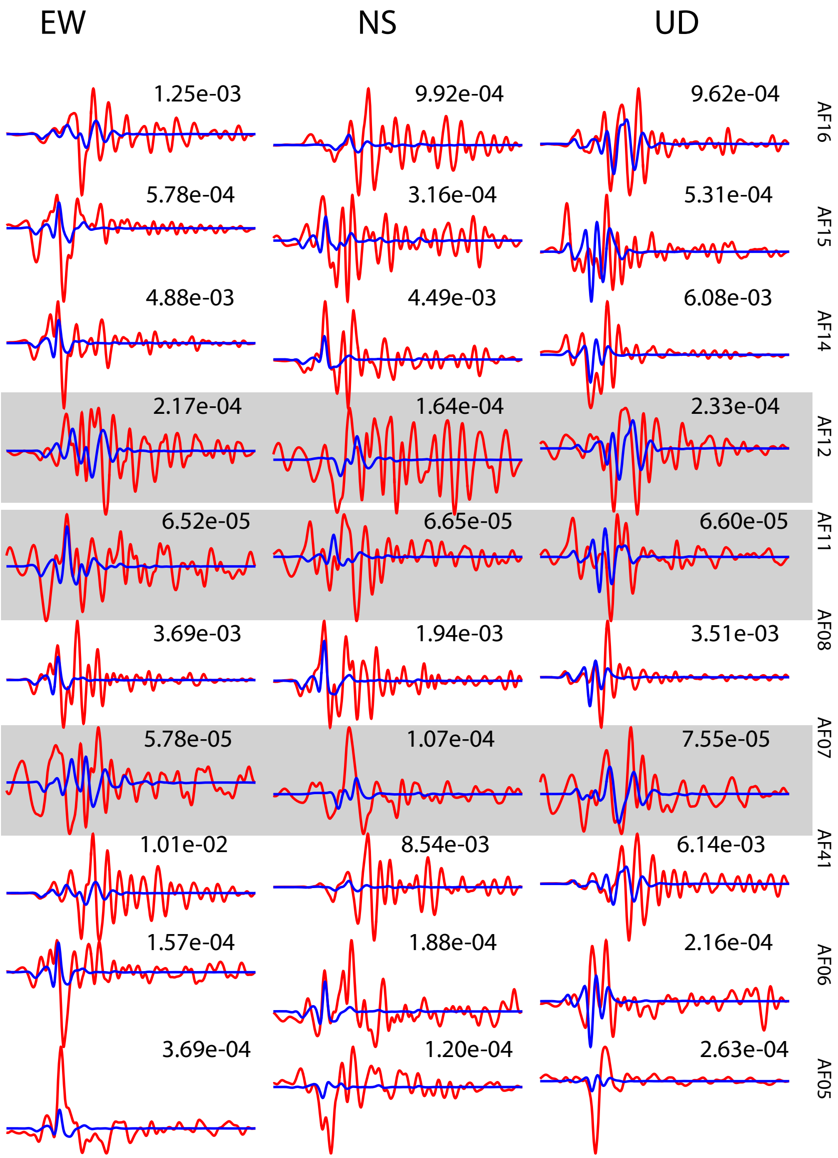
Covariance of green function errors – test case L'Aquila

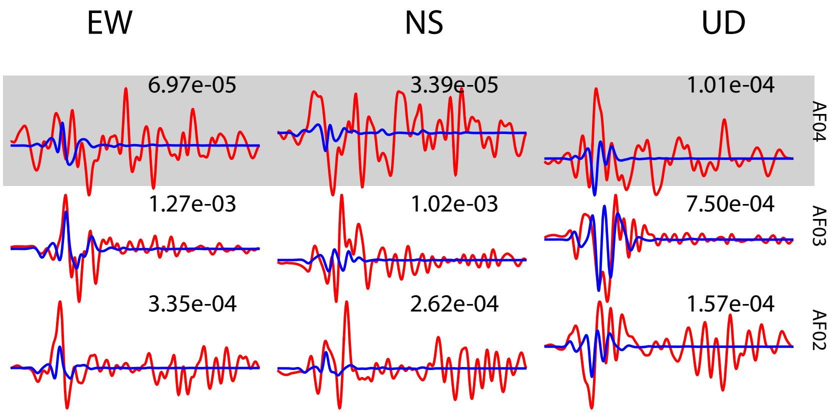
Observed ground velocity at **FIAM** (red) and synthetic velocity (blue) for the CIA structure, plotted at the same scale as the observations, for a subset of the aftershocks. Number is peak velocity of data seismogram. First 40s of total 60s seismograms are shown.



Covariance of green function errors – test case L'Aquila

Observed ground velocity at **FIAM** (red) and synthetic velocity (blue) for the CIA structure, plotted at the same scale as the observations, for a subset of the aftershocks. Number is peak velocity of data seismogram. First 40s of total 60s seismograms are shown.





The moment-tensor solutions were obtained by fitting lower frequency data at more distant stations. Thus, fits shown in these figures are not the result of inversions of the observed aftershock data but rather of forward modeling of these events at frequencies up to 0.5 Hz.

Recovering traction from ground velocity

For each frequency and component of motion we form the complex difference

$$\Delta_i^j = v_i - s_i^j$$

s_i is the aftershock synthetic

v_i is the observed aftershock datum, which is the product of a moment times the Green's function divided by a rigidity.

$$v_i = \frac{M_i}{\mu_i} g_i$$

The ***complex difference*** for each frequency and component is then

$$\Delta_i^j = v_i - s_i^j = v_i - \tilde{P}_i^j \, \tilde{g}_i^j \, .$$

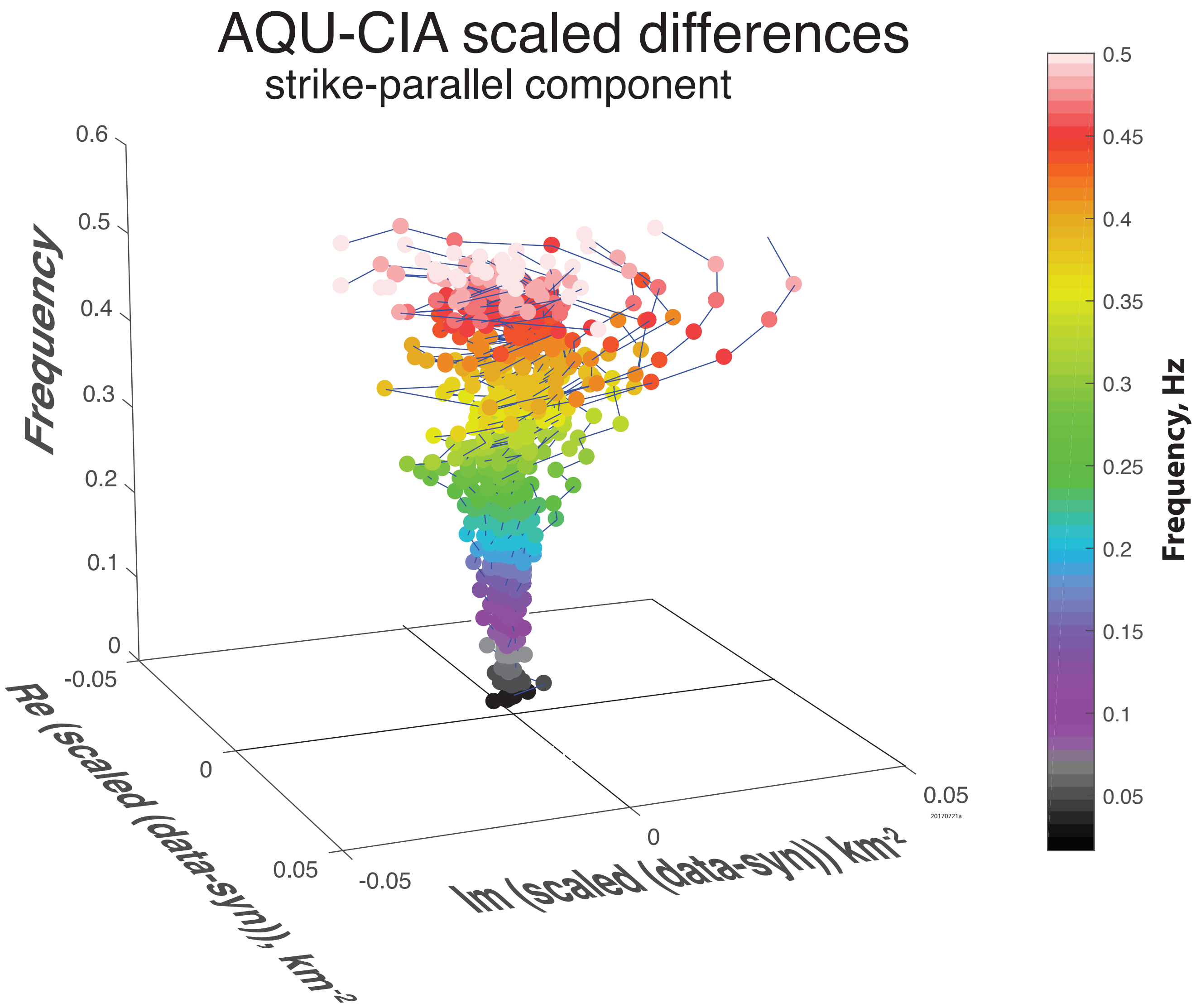
Normalizing by seismic moment and rigidity yields a quantity with the units of the traction Green's function, namely the ***scaled complex difference*** (or equivalently the ***empirical traction error***)

$$\tilde{\Delta}_i^j = \left(c \, \Delta_i^j / \tilde{P}_i^j \right) = \left(\frac{v_i}{\tilde{P}_i^j} - \tilde{g}_i^j \right) c$$

Covariance of green function errors – test case L'Aquila

Empirical traction errors differences in the complex plane for **AQU** station using **CIA model**

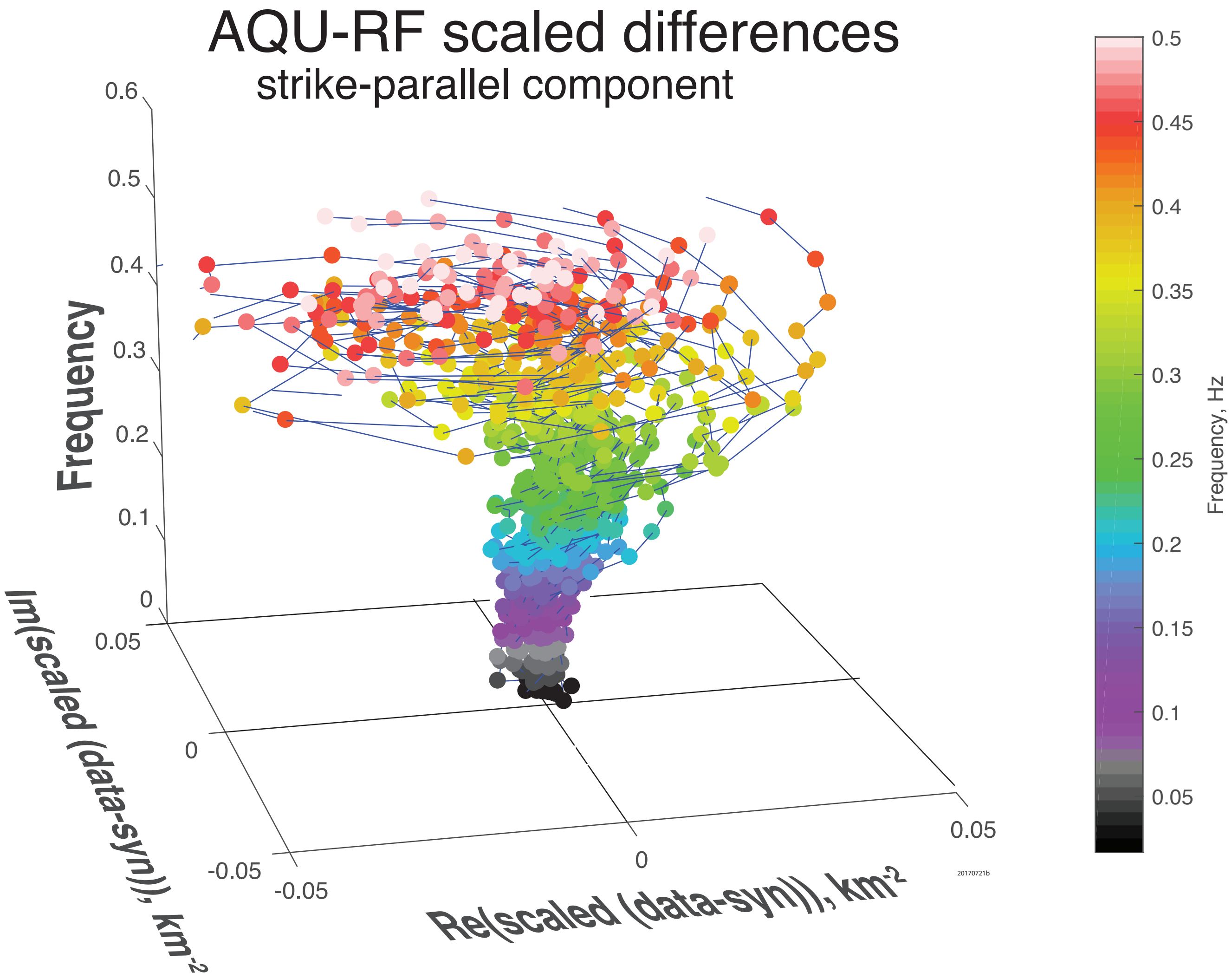
They show that the empirical traction errors grow in magnitude as frequency increases, justifying the appropriateness of our frequency-domain approach.



Empirical traction errors differences in the complex plane for **AQU** station using **RF model**

For many aftershocks the empirical traction errors form expanding helices, corresponding to progressive phase shifts as a function of frequency

we have not introduced 'static corrections' into the theoretical Green's functions to remove these time mismatches because time mismatches are errors in the theoretical Green's functions, the effect of which we hope to quantify



Covariance of green function errors – test case L'Aquila

The covariance between the empirical traction at x_i and x_k for component of motion j and frequency index n is

$$K_{ik}^{j}\left(\omega_n\right)=C_{11}^{j}\left(\mathbf{x}_i,\mathbf{x}_k,\omega_n\right)=E\left[\delta g_1^{j}\left(\mathbf{x}_i,\omega_n\right)^{*}\delta g_1^{j}\left(\mathbf{x}_k,\omega_n\right)\right]=E\left[\tilde{\Delta}^{j}\left(\mathbf{x}_i,\omega_n\right)^{*}\tilde{\Delta}^{j}\left(\mathbf{x}_k,\omega_n\right)\right]$$

We omit the slip direction indices entirely as the aftershock rakes are chosen to be within 30 degrees of the dominant slip direction.

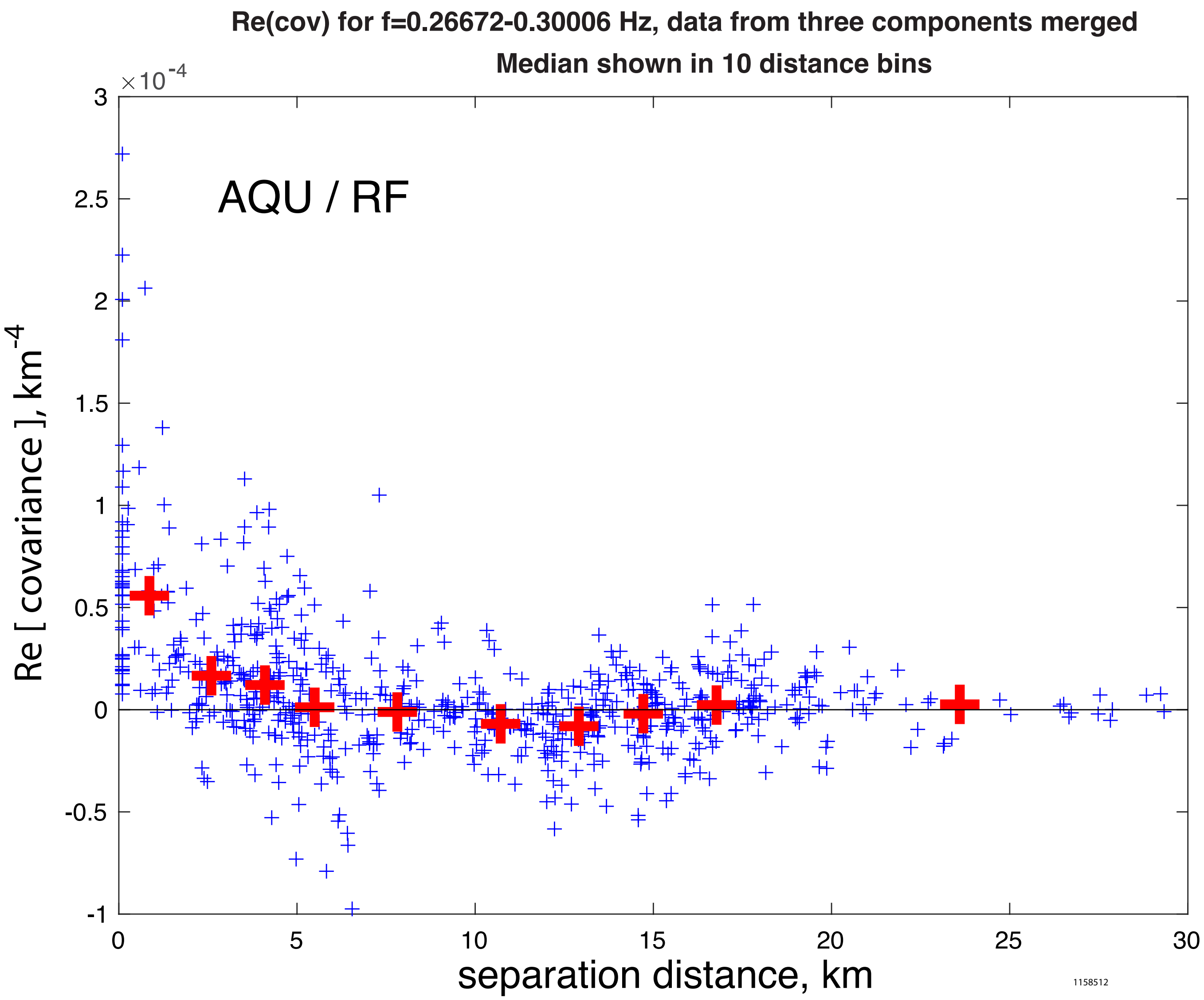
To take the expected value, we average the above over all three components of motion j and over a frequency band of three adjacent frequencies n:

$$C\left(\mathbf{x}_i,\mathbf{x}_k,\omega_{n_o}\right)=\frac{1}{3}\sum_{j=1}^{3}\frac{1}{3}\sum_{n=n_o}^{n_o+2}\mathrm{Re}\left(\hat{\Delta}^{j}\left(\mathbf{x}_i,\omega_n\right)^{*}\hat{\Delta}^{j}\left(\mathbf{x}_k,\omega_n\right)\right)$$

We call each value of $C\left(\mathbf{x}_i,\mathbf{x}_k,\omega_{n_o}\right)$

a covariance datum, and in the next slide we plot all the covariance data for a single station and frequency band n_o as a function of separation $r=\left|\mathbf{x}_i-\mathbf{x}_k\right|$

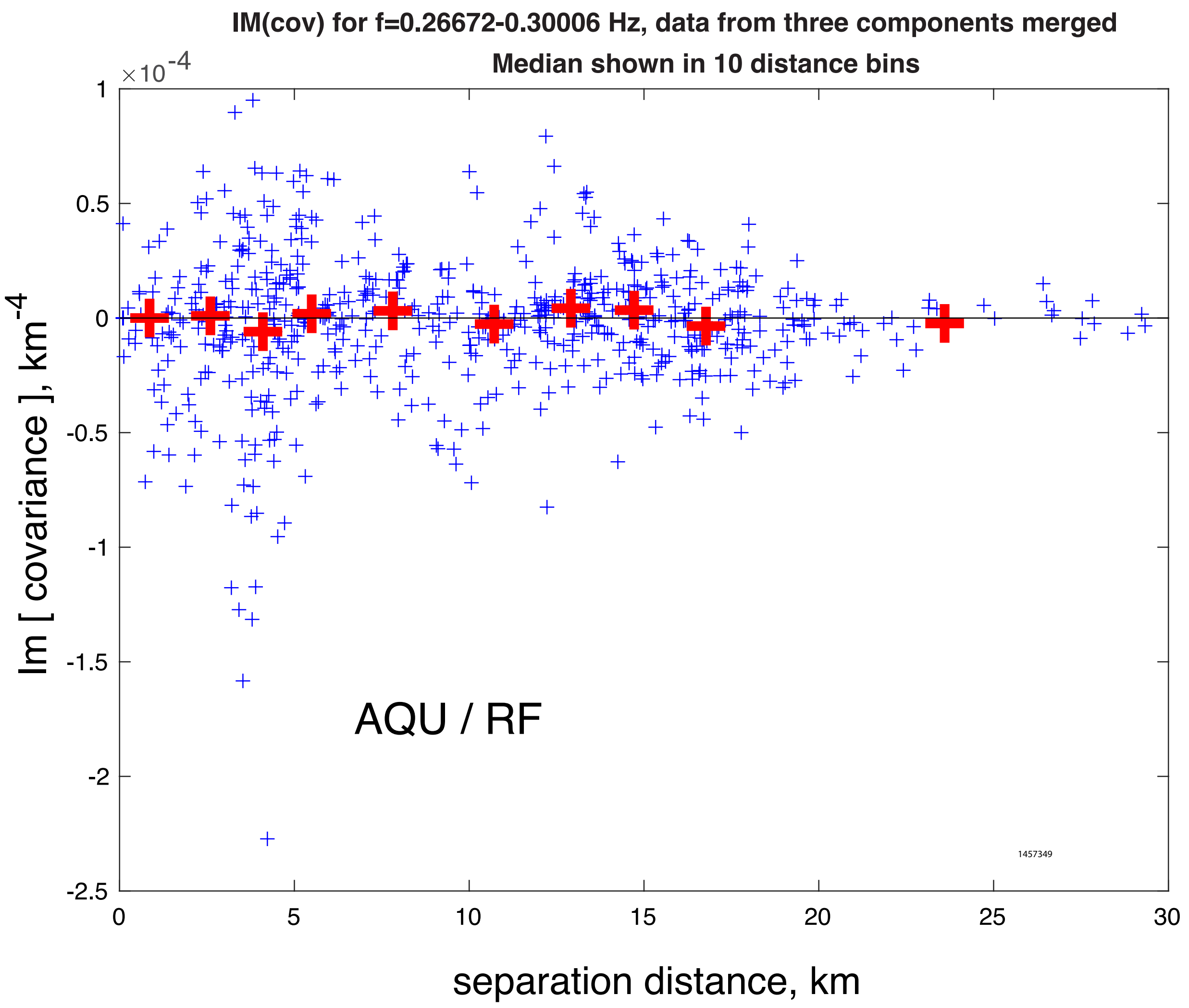
Covariance function from empirical traction errors – test case L'Aquila



Blue dots: Example of the real part of the Covariance data for a fixed small frequency band.

Red crosses: median values in each of 10 separation distance bins

Covariance function from empirical traction errors – test case L'Aquila

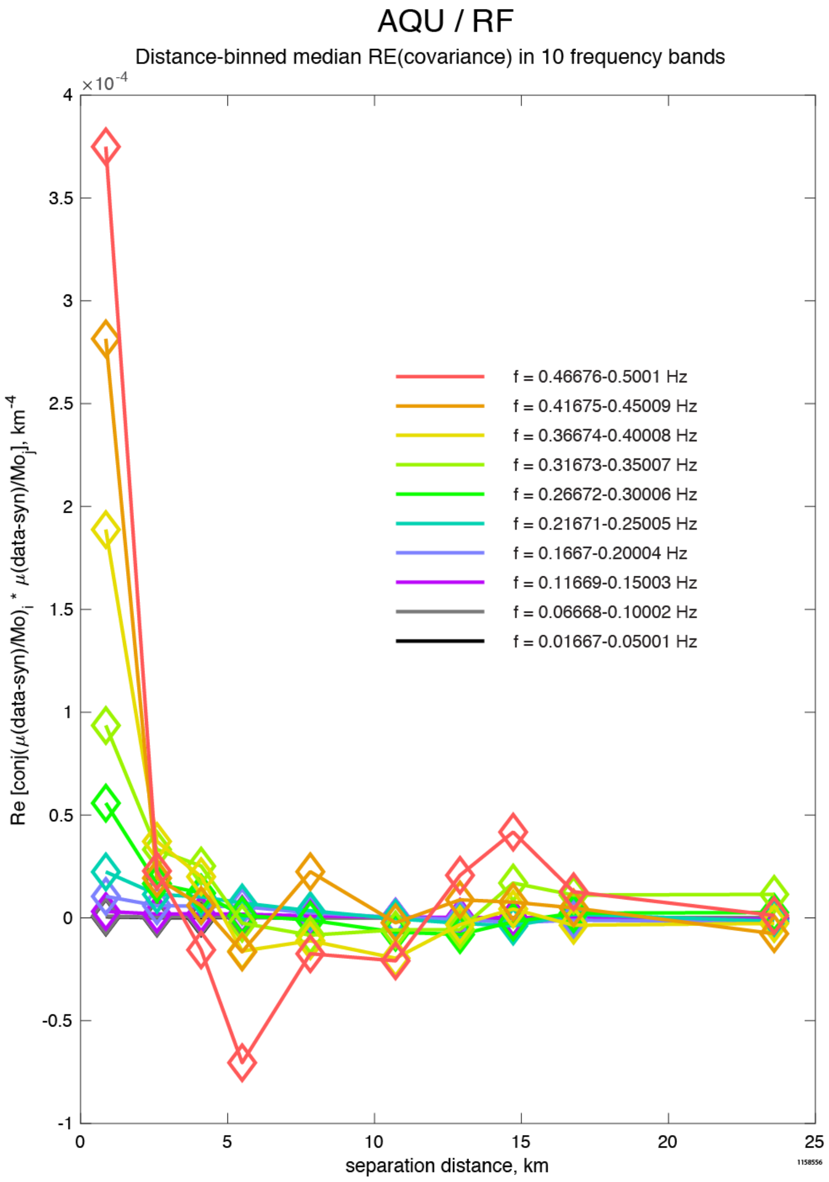


The median of the imaginary part of the covariance data is not significantly different from zero

Covariance function from empirical traction errors – test case L'Aquila

AQU COVARIANCE FUNCTIONS FOR ALL STUDIED FREQUENCIES

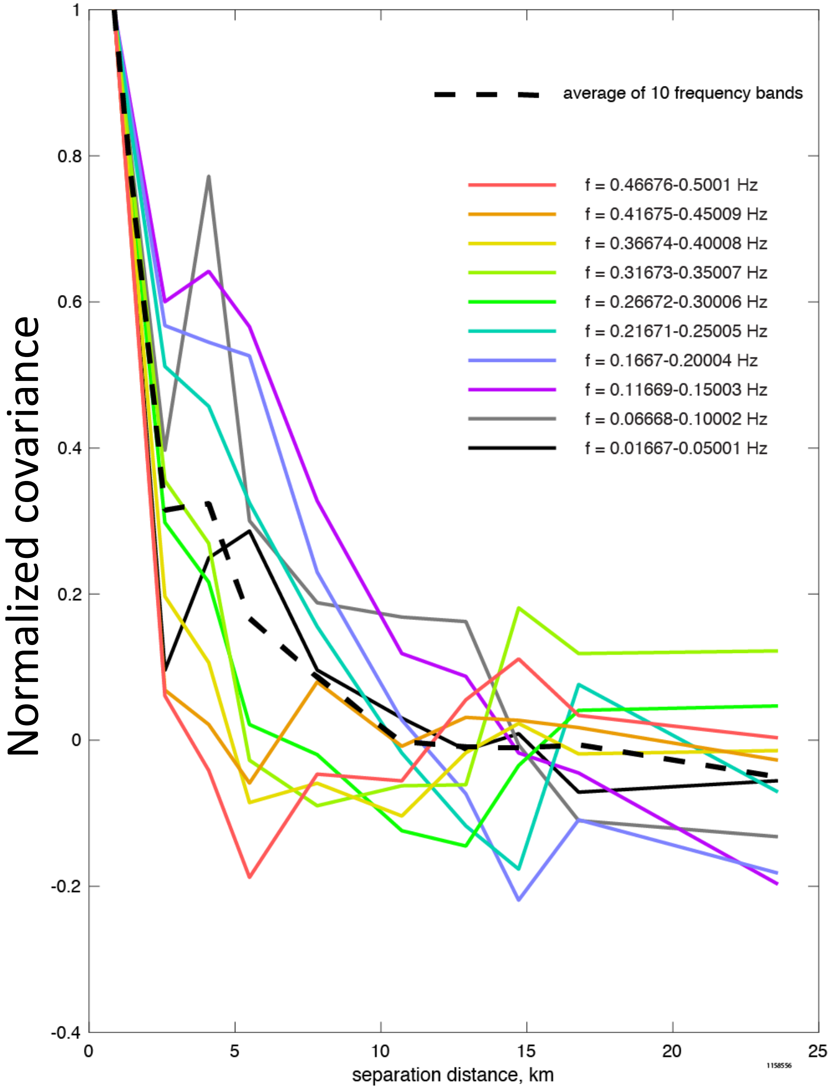
Covariance functions for 10 frequency bands at AQU using the RF velocity structure



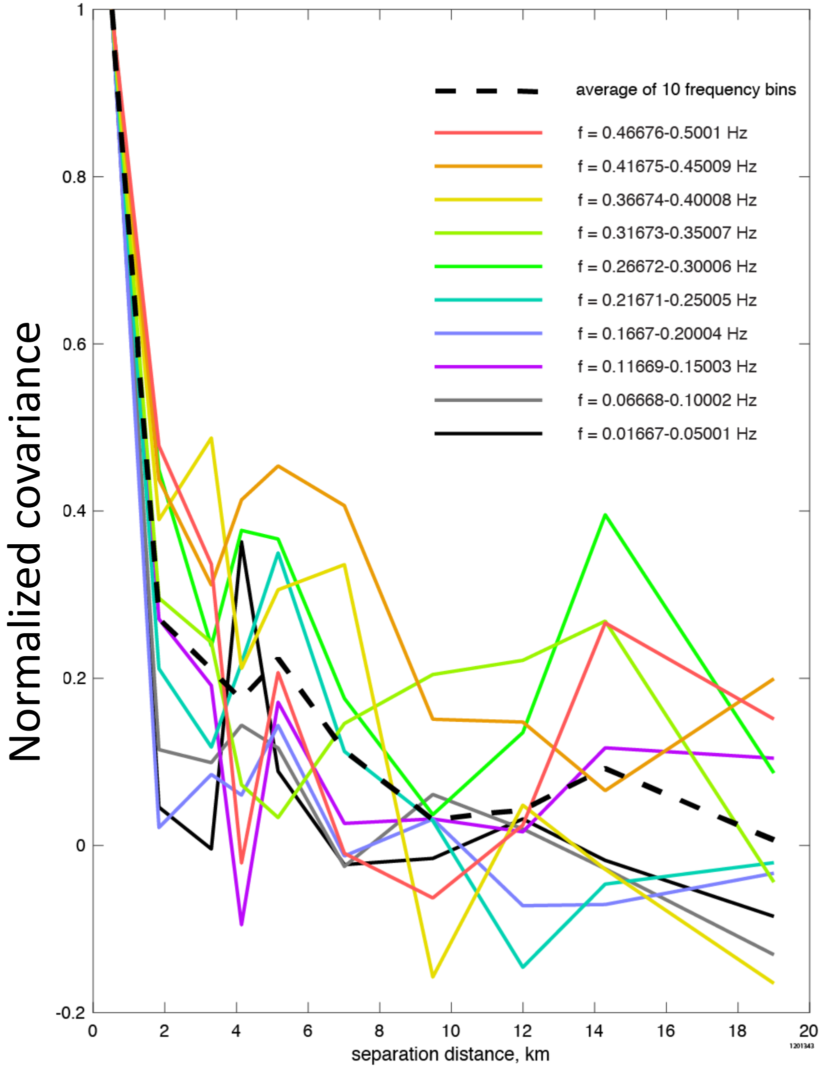
Covariance function from empirical traction errors – test case L'Aquila

NORMALIZED COVARIANCE FUNCTIONS FOR ALL STUDIED FREQUENCIES

AQU + RF model

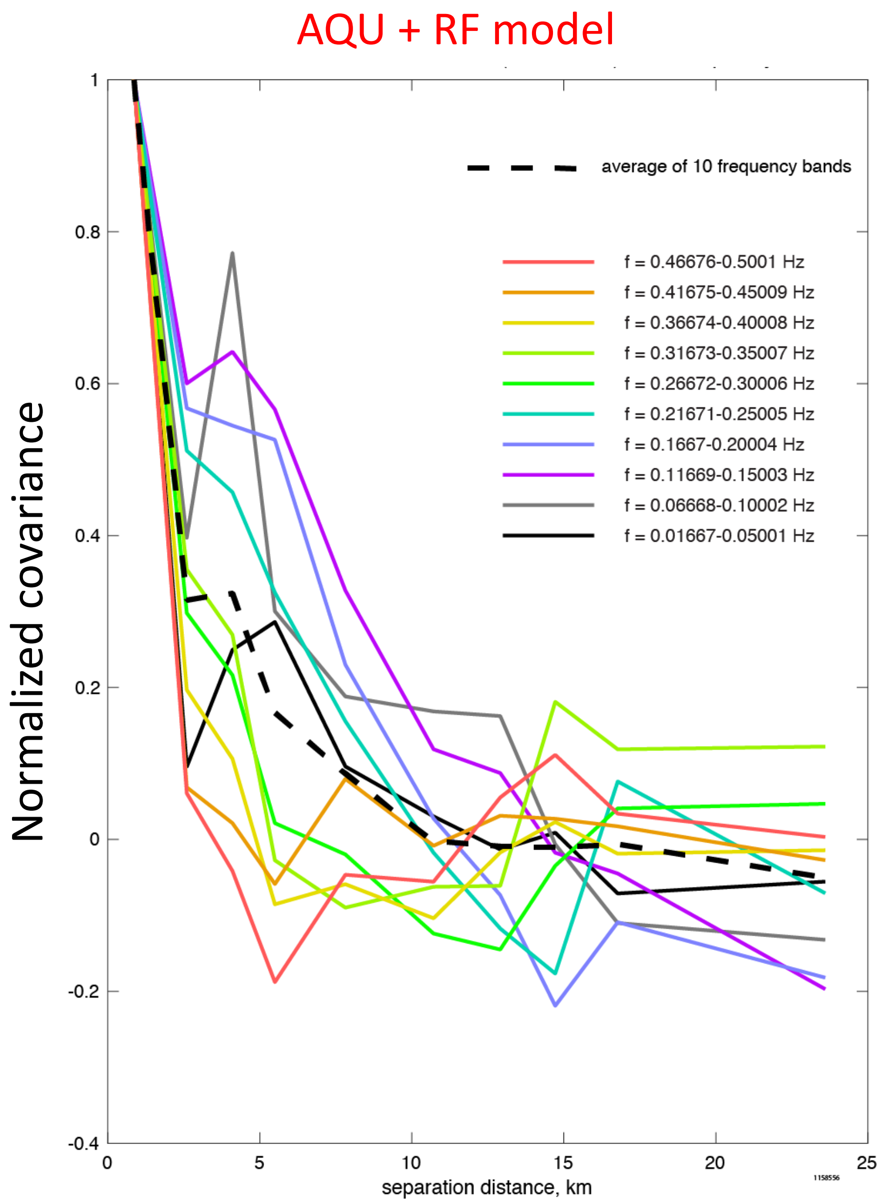


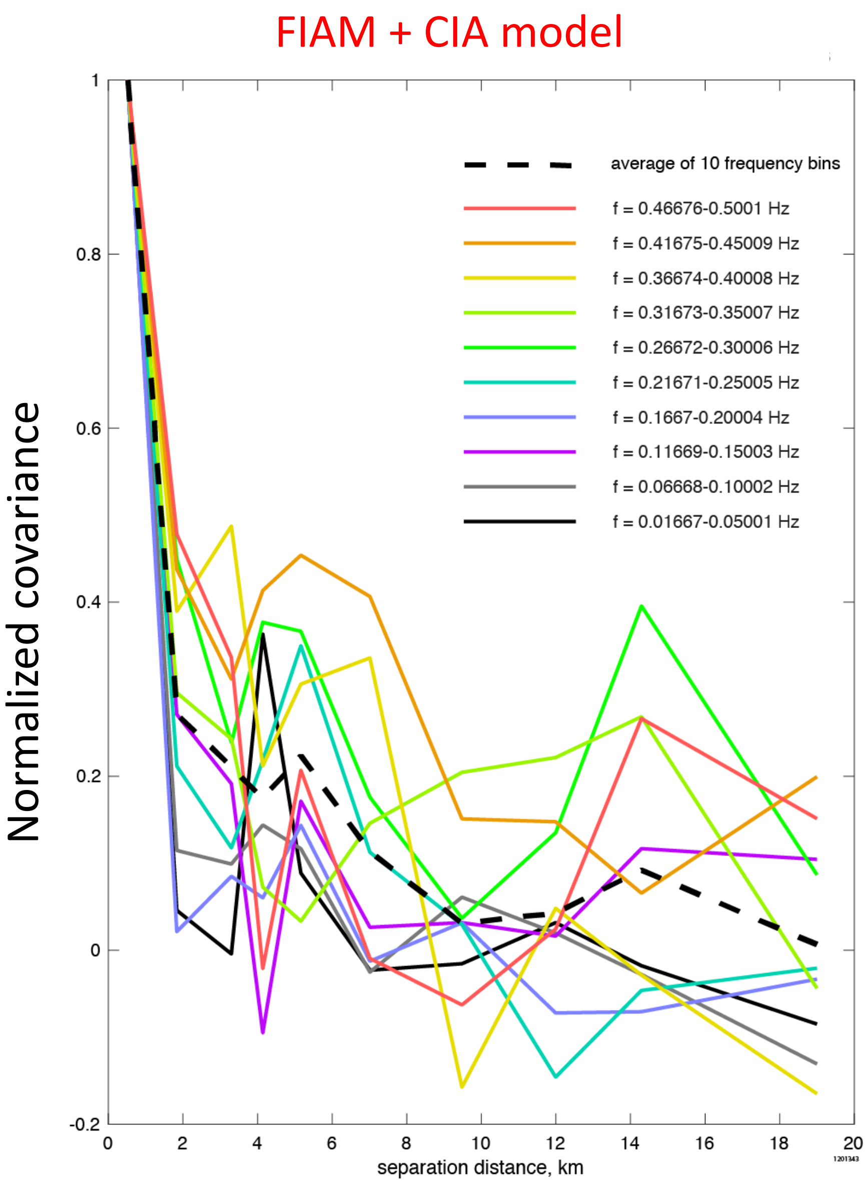
FIAM + CIA model



The dashed average covariance functions are similar in shape to Frankel and Clayton (1986) covariance functions for exponential and self-similar media

Covariance function from empirical traction errors – test case L'Aquila





no frequency dependence is seen on the right panel for FIAM

Variance– test case L'Aquila

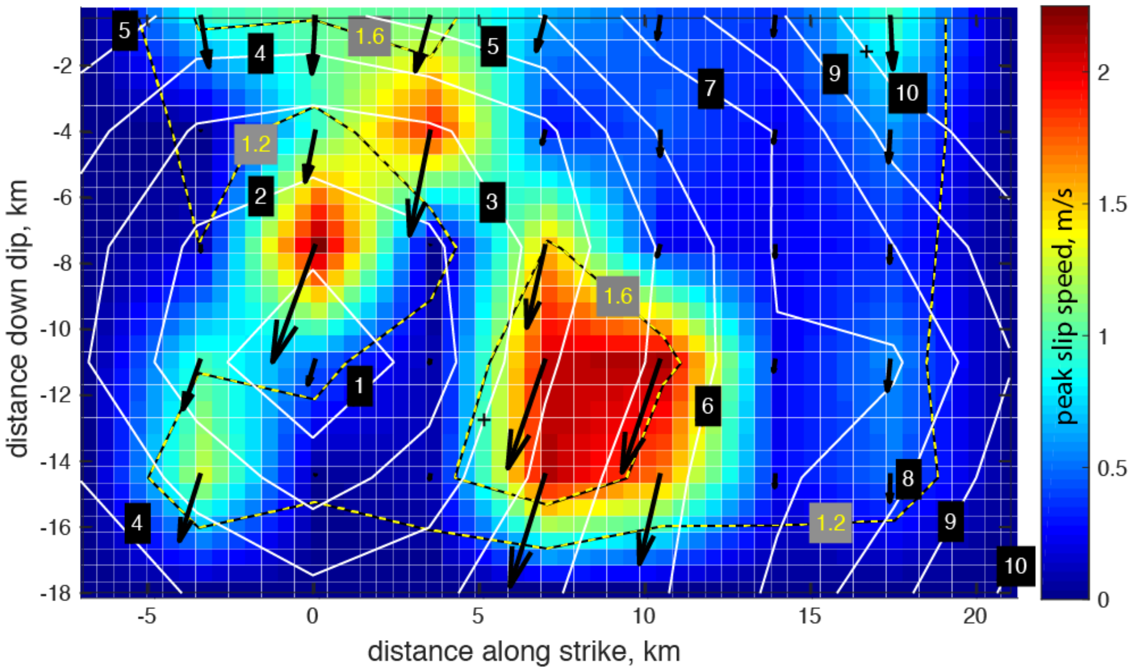
We evaluate γ^2 for a specific rupture model of the L'Aquila earthquake, namely the minimum cost model found by Cirella et al. (2009)

$$\gamma_{sc}^2(\omega) = \int_{\mathbf{x}\in A} \int_{\mathbf{x}'\in A} \tilde{s}_1^*(\mathbf{x},\omega)\, K^{sc}(r,\omega)\, \tilde{s}_1(\mathbf{x}',\omega)\, d\mathbf{x}\, d\mathbf{x}'$$

where

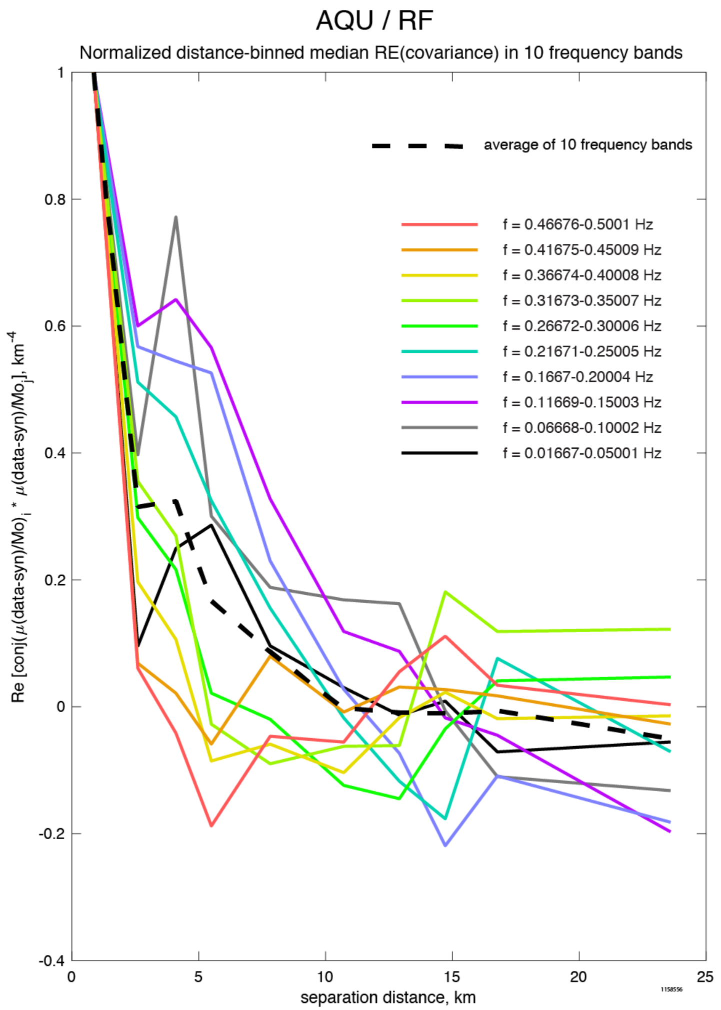
- j is the channel number, a single component of motion at a single station
- A is the rupture area.

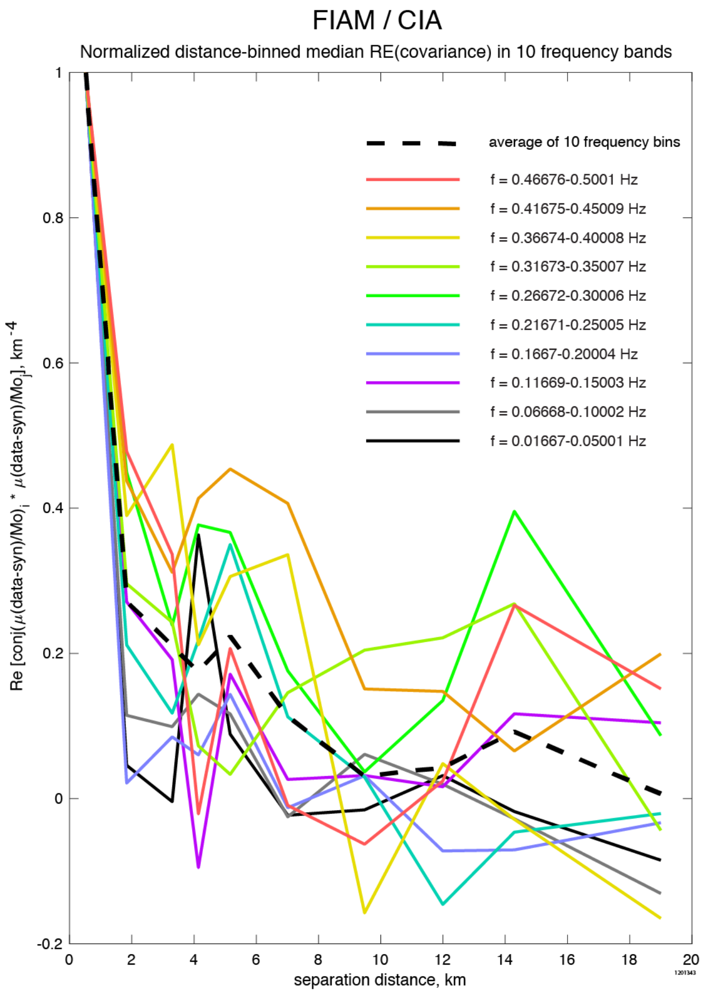
Best fitting L'Aquila rupture model



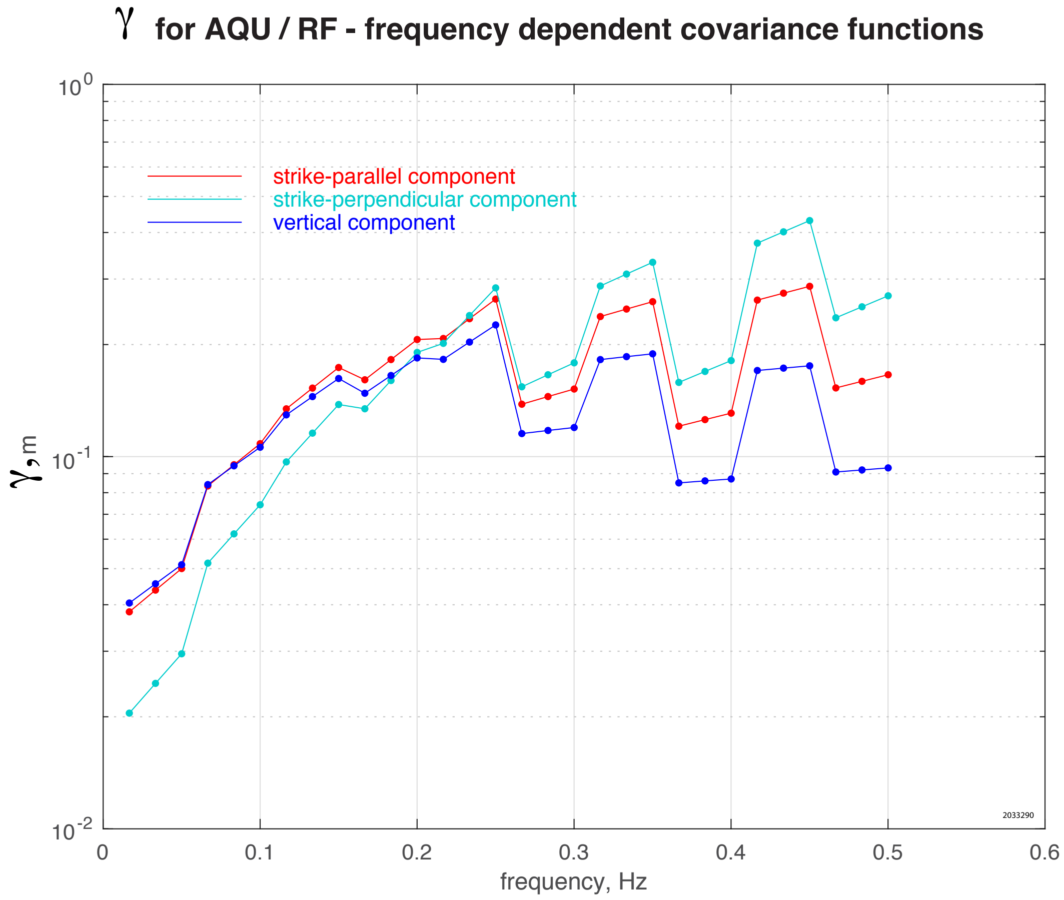
Variance– test case L'Aquila

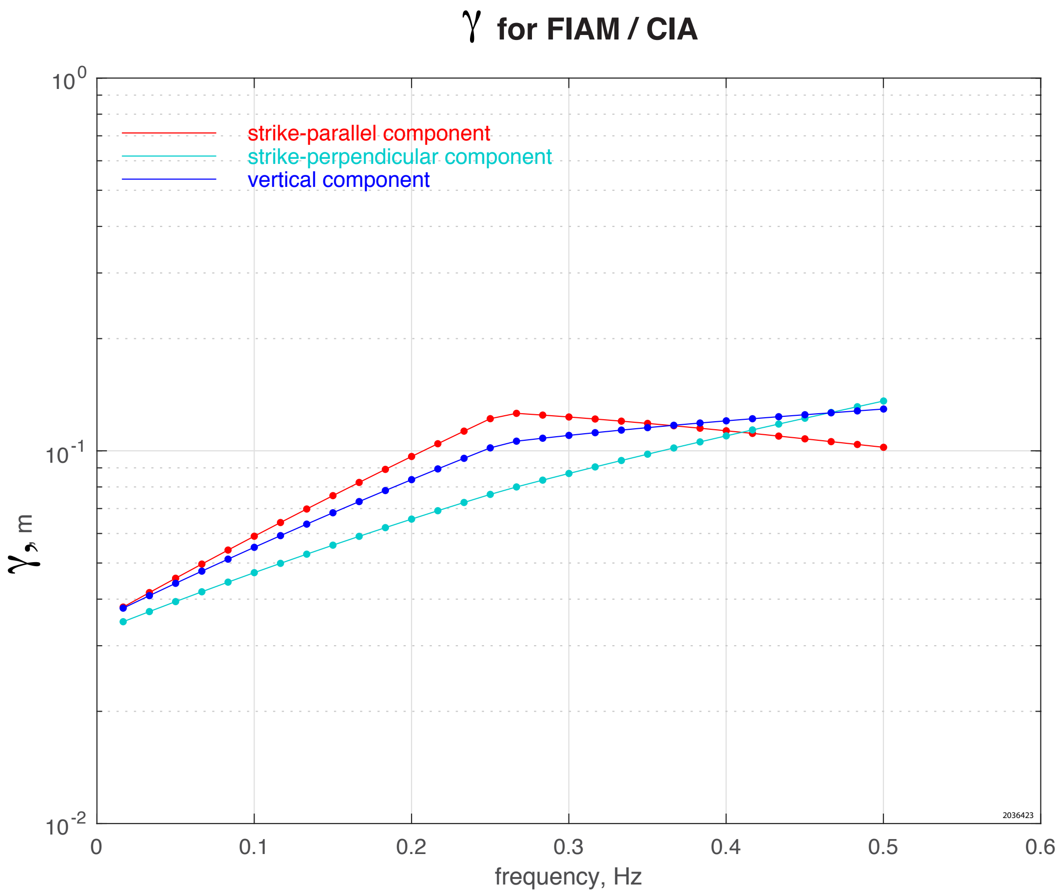
Reminder: Because for AQU / RF there was a possible frequency dependence to the covariance curves, we used each individual colored curve as the covariance function AQU for its associated frequency band. For FIAM we used the dashed average covariance function for all frequencies.





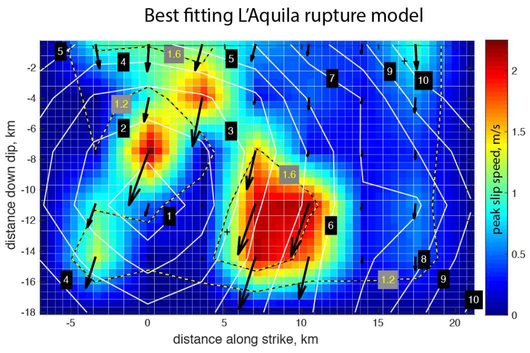
γ spectra for AQU / RF and FIAM / CIA, all components



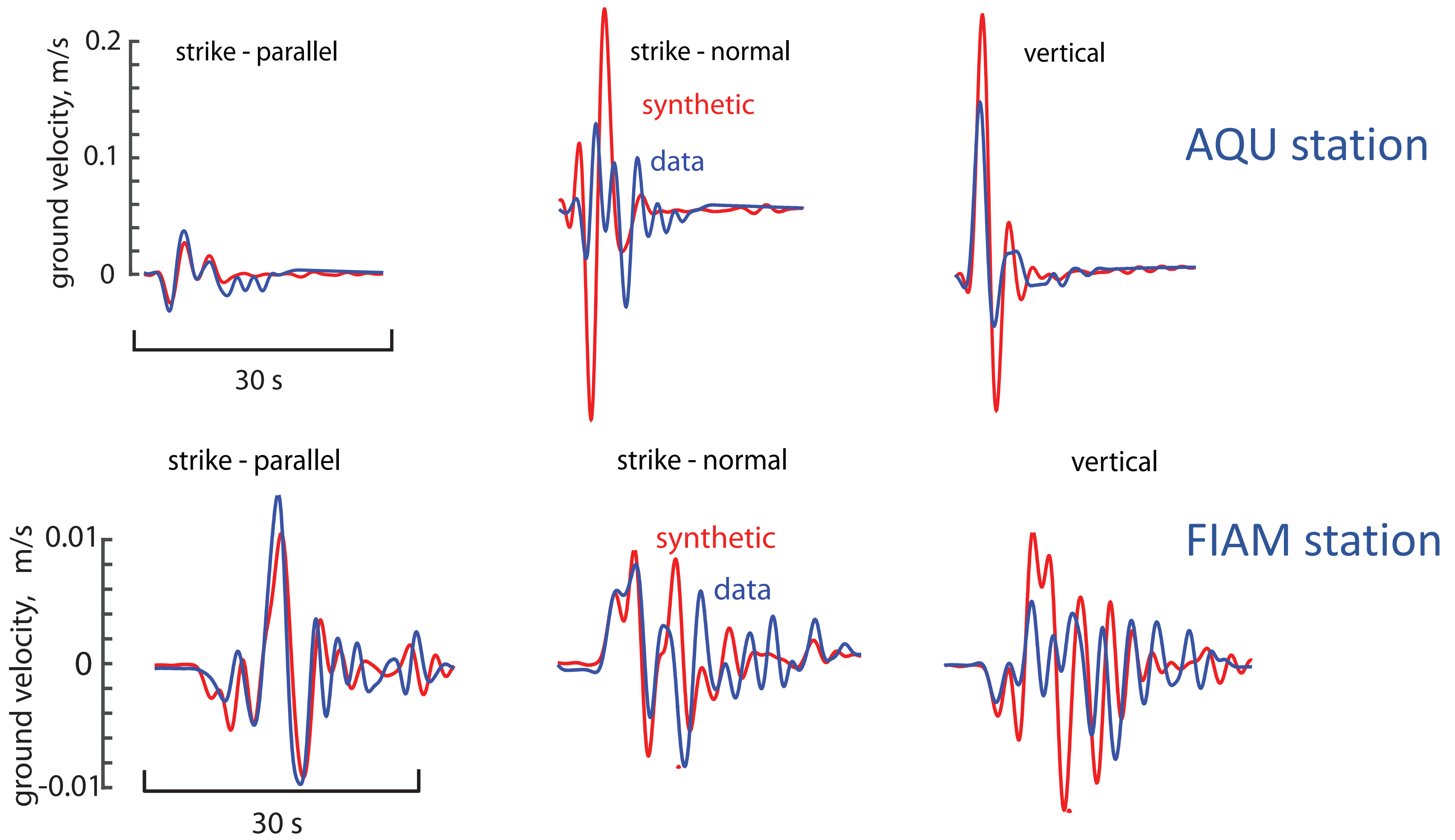


Variance– test case L’Aquila

Now we can compare the main shock data-minus-synthetic misfit with γ, the misfit expected from green’s function errors.

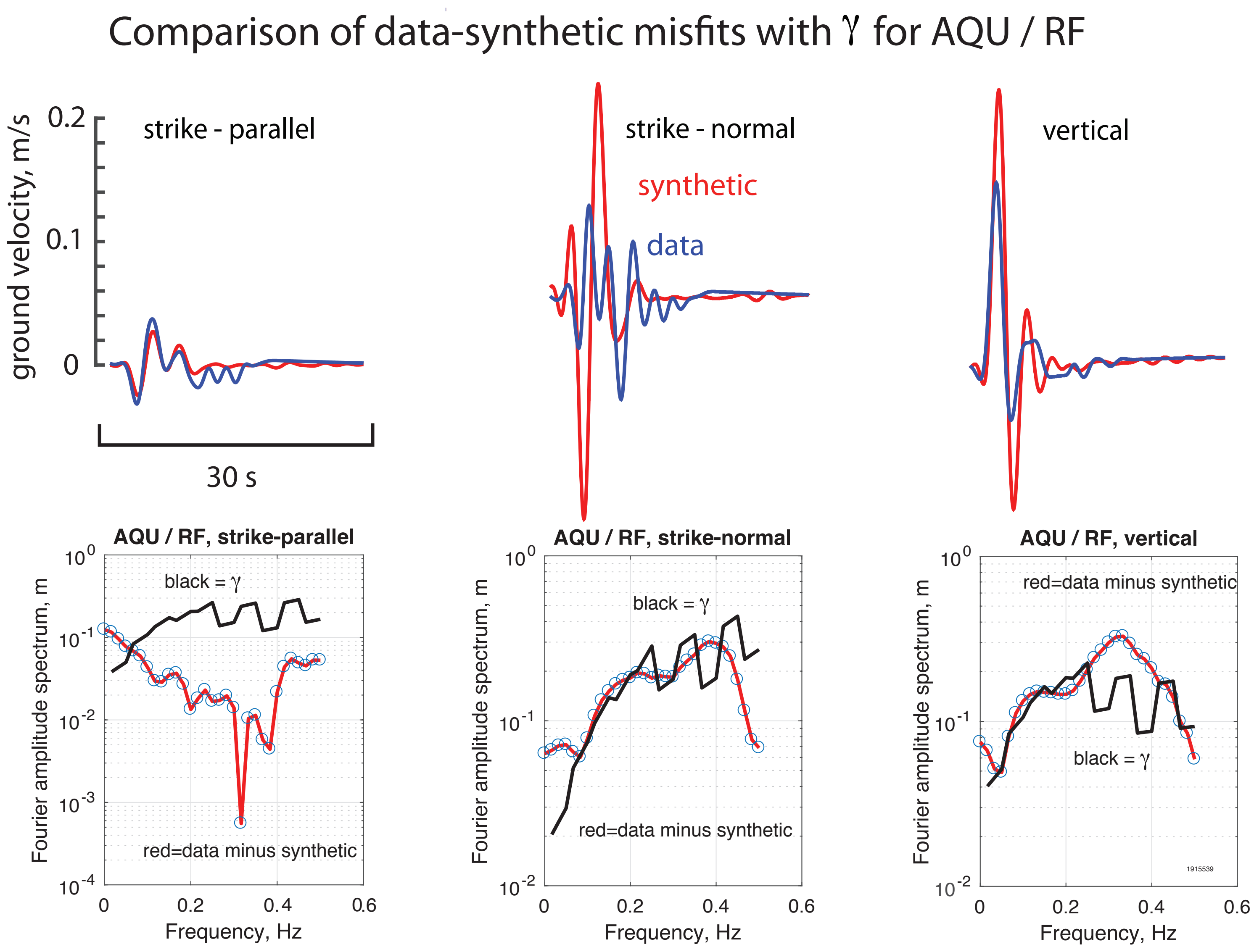


Fit of the Mw=6.1 2009 L’Aquila event for frequency 0.02-0.5 Hz



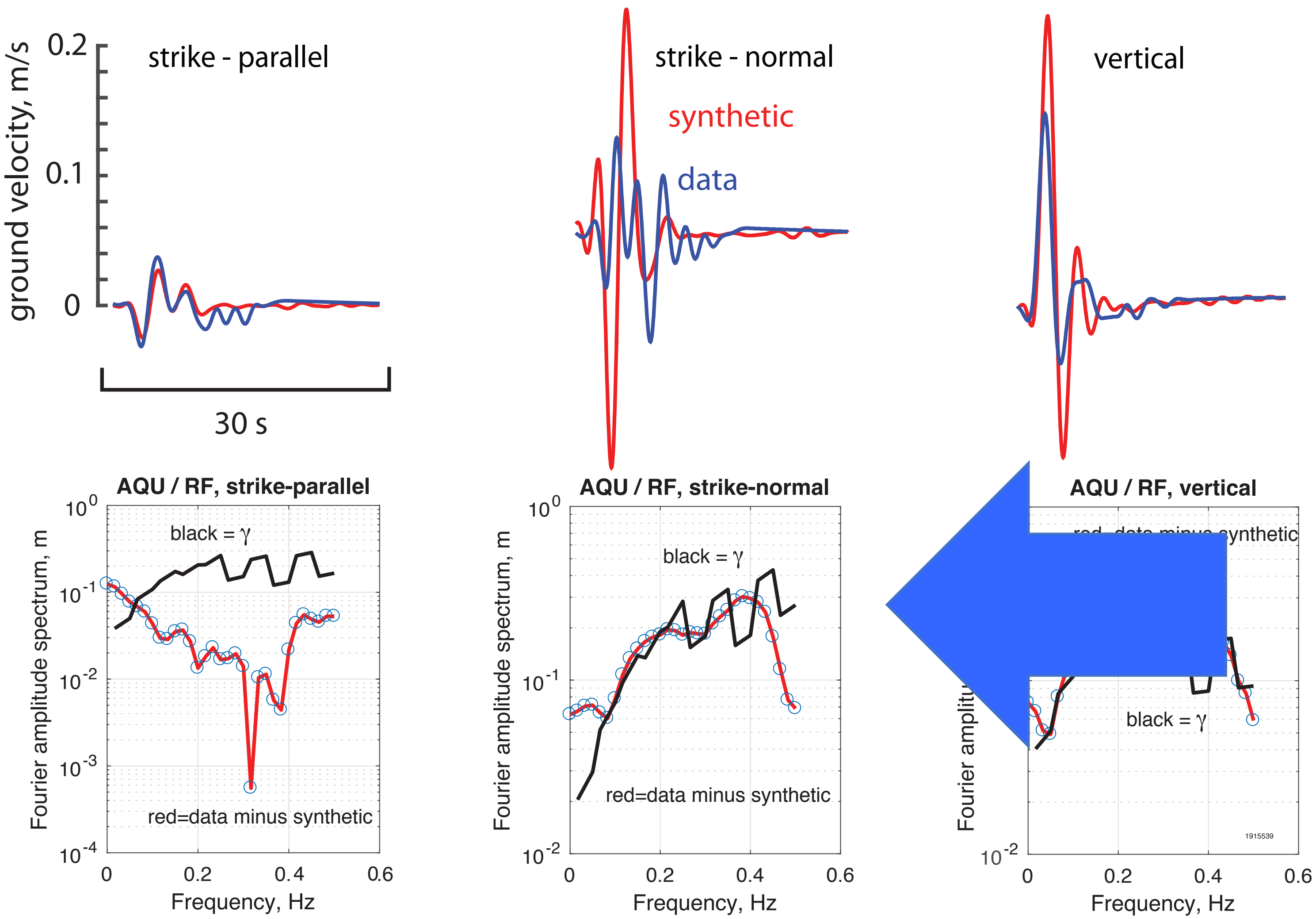
Variance– test case L'Aquila

AQU station



AQU station

Comparison of data-synthetic misfits with γ for AQU / RF

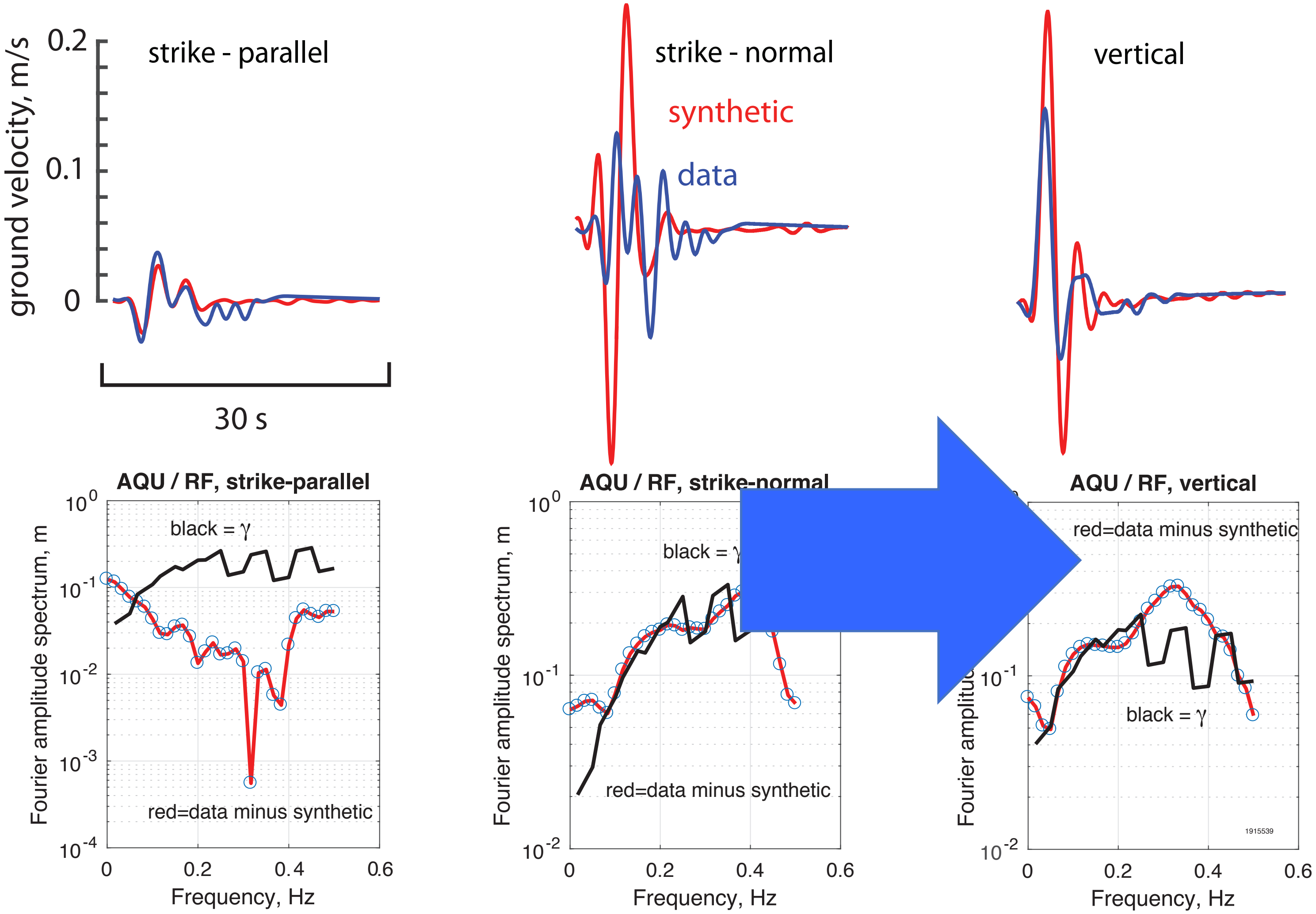


The agreement of the γ spectrum with the red misfit tells us that the misfit we see in the seismogram is consistent with the misfit expected

Variance– test case L'Aquila

AQU station

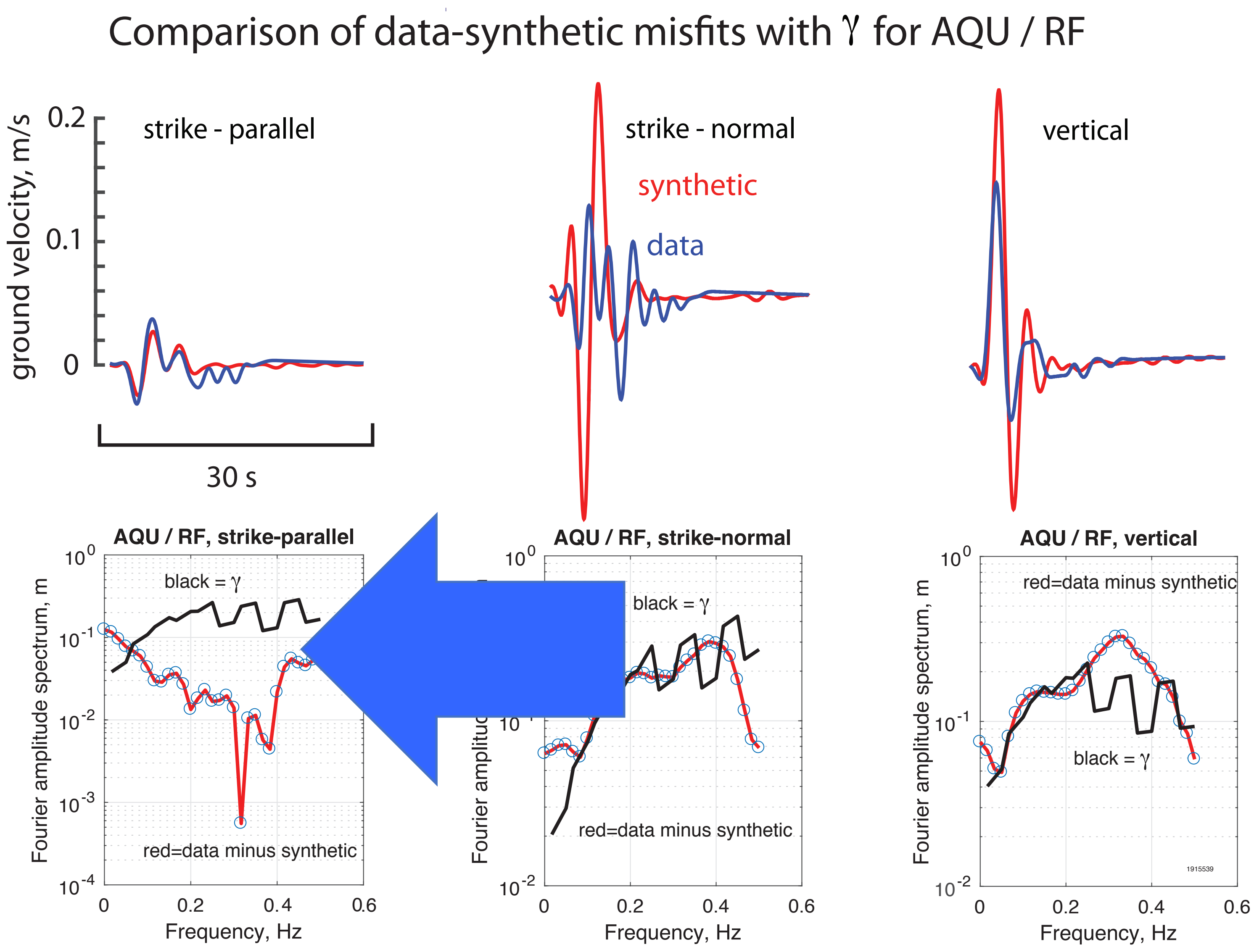
Comparison of data-synthetic misfits with γ for AQU / RF



The γ spectrum lies below the red misfit from 0.25 – 0.45 Hz: the seismogram is undefit.

Variance– test case L'Aquila

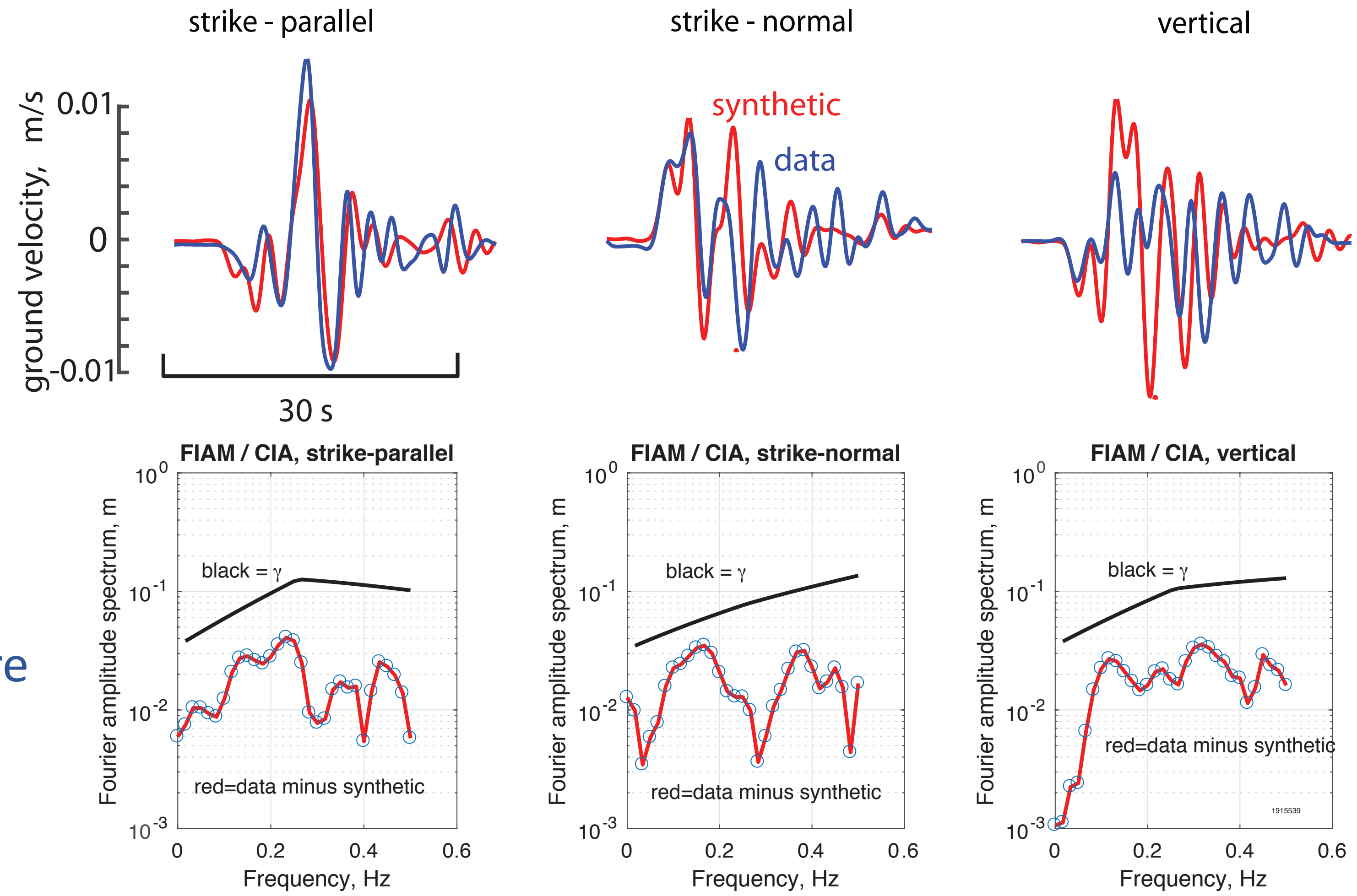
AQU station



Strike – parallel component are over-fit: it shows an excessively good fit to the data considering the non-negligible theory errors.

Variance– test case L'Aquila

Comparison of data-synthetic misfits with γ for FMG / CIA



All the components are over-fit

Two other sources of variability in earthquake ground velocity:

$$v_j(\omega, \mathbf{y}) = \int_{\mathbf{x}\in A} \tilde{\mathbf{s}} \bullet \tilde{\mathbf{g}}^{(j)} dA + \int_{\mathbf{x}\in A} \delta\mathbf{s} \bullet \tilde{\mathbf{g}}^{(j)} dA + \int_{\mathbf{x}\in A} \tilde{\mathbf{s}} \bullet \delta\mathbf{g}^{(j)} dA + \int_{\mathbf{x}\in A} \delta\mathbf{s} \bullet \delta\mathbf{g}^{(j)} dA.$$

the ground velocity caused by perturbations of the rupture model

the ground velocity caused by joint perturbations of the rupture model and errors in the Green's functions

The variability in ground velocity of the j-th channel (single component of motion at a particular observation location) caused by perturbations of the rupture model has the following variance:

$$\delta v_j^s = \int\limits_{\mathbf{x} \in A} \delta \mathbf{s} \bullet \tilde{\mathbf{g}}^{(j)} dA$$

$$\boxed{\rho_j^2(\omega) = \int\limits_{\mathbf{x} \subset A} \int\limits_{\mathbf{x}' \subset A} \tilde{g}_1^{(j)*}(\mathbf{x}, \omega)\, S_{11}(\mathbf{x}, \mathbf{x}', \omega)\, \tilde{g}_1^{(j)}(\mathbf{x}', \omega)\, d\mathbf{x}\, d\mathbf{x}'}$$

It depends on the spatial covariance of perturbations of the rupture model S_{11}

Future applications

- The primary use will be in seismic hazard studies to calculate the variability of synthetic seismograms given an ensemble of rupture models
- The variance of the ground motion is determined directly, skipping the step of calculating the ground motions of many rupture models.

Variability of the ground velocity due to the interaction between δs and δg has the following variance:

$$\delta v_j^{sg} = \int_{\mathbf{x}\in A} \delta\mathbf{s} \bullet \delta\mathbf{g}^{(j)} dA = \int_{\mathbf{x}\in A} \left(\delta s_1\, \delta g_1^{(j)} + \delta s_2\, \delta g_2^{(j)}\right) dA$$

$$\xi_j^2(\omega) = \int_{\mathbf{x}\in A} \int_{\mathbf{x}'\in A} Cov\left[\delta s_1(\mathbf{x}')\, \delta g_1^{(j)}(\mathbf{x}'),\ \delta s_1(\mathbf{x})\, \delta g_1^{(j)}(\mathbf{x})\right] d\mathbf{x}\, d\mathbf{x}'$$

Future applications

The possibility of a nonzero covariance between δs and δg in the error interaction term opens an interesting line of research. We can imagine that spatial variations of rigidity in the fault zone might cause spatial variations of δg. These spatial variations of rigidity might also cause correlated variations in the rupture process δs. It would be very interesting to look for such correlations in numerical simulations of spontaneous rupture in heterogeneous media.

CONCLUSIONS –

We have found a simple equation relating the variance in the ground motions predicted from a given slip model to the spatial covariance function of the Green's function errors.

This variability would be considered to be epistemic, as it is caused by unknowns in the Earth structure, which could be improved by collection of more data.

The spatial covariance function of Green's function errors can be recovered from analysis of small earthquakes (like aftershocks) spanning the rupture surface.

For regions with sparse seismicity, it might be possible to define a spatial covariance function from study of teleseismic amplitude and travel-time variations, and from coda-Q.

CONCLUSIONS –

We have computed the expected variance (and the standard deviation) of ground motion variations due to Green's function errors for the Mw=6.1 2009 L'Aquila earthquake;

We have compared the inferred standard deviation with the misfit of synthetic and real data for the Mw=6.1 2009 L'Aquila earthquake and we have discussed/discovered which are the data over-fitted by the slip model.

Thanks

***Paul Spudich, Antonella Cirella, Laura Scognamiglio, Elisa Tinti**, (2'019) Variability in synthetic earthquake ground motions caused by source variability and errors in wave propagation models, Geophysical Journal International, ggz275, https://doi.org/10.1093/gji/ggz275*