Workshop on Numerical Modeling of Earthquake Motions: Waves and Ruptures, Bratislava, Slovakia, July 1-4, 2019

Equation of Motion for Dynamic Rupture Pulses
(why pulse may be the dominant rupture mode on mature faults)

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Seismological observations tend to indicate that major earthquake ruptures propagate along seismogenic faults pulse-like [Heaton PEPI 1990], such that any given point of the ruptured fault accumulates slip over duration much shorter than the overall fault rupture time. Numerical simulations of elastodynamic rupture on strongly dynamically weakening faults [Gabriel et al JGR 2012, Noda et al JGR 2009] have shown that slip pulses do arise spontaneously, but generally are inherently transient even on uniformly-stressed, homogeneous faults - either arresting or accelerating to the limiting speed, prompting transition to crack-like and/or supershear mode.

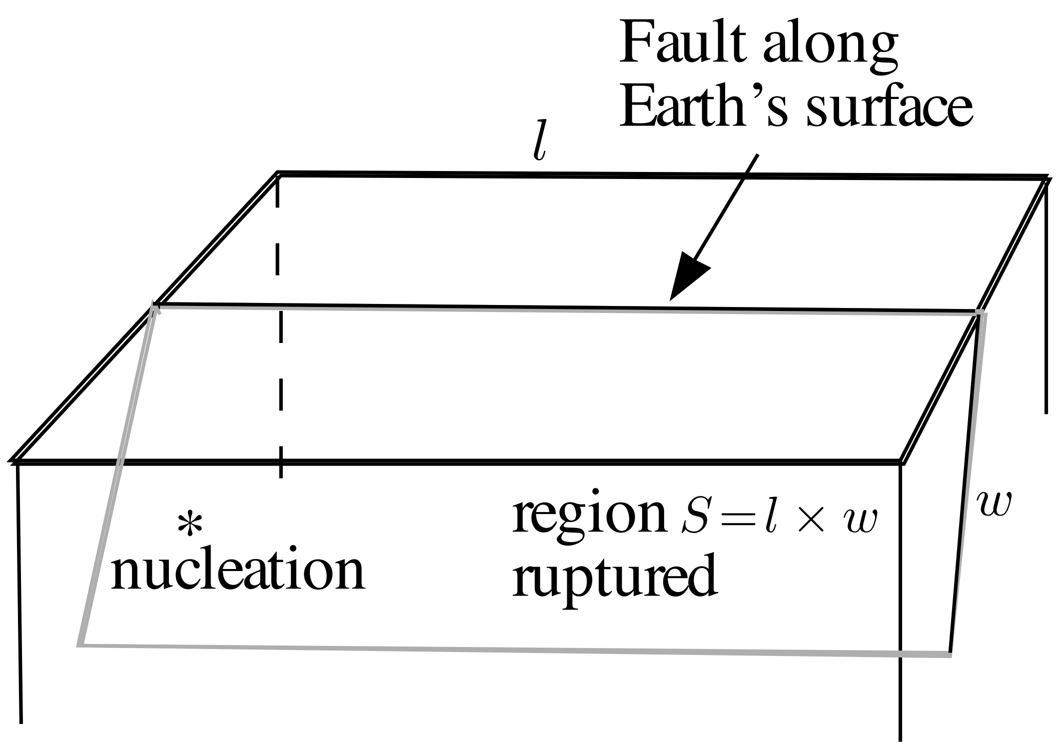
In this work, we have developed an equation of motion (EoM) for transient pulse motion based on the analysis of the fault field perturbation around a steadily propagating pulse solution. The latter can be readily obtained and fully parametrically characterized for a given fault rheology (see, e.g., Garagash [JGR 2012] for steady pulses driven by thermal pressurization). We validate the pulse EoM by full elastodynamics rupture simulations in the case of a fault dynamically weakened by thermal pressurization (TP).

The rate of pulse slip in the EoM is proportional to the difference between the actual background shear stress resolved on the fault plane and the "steady-state pulse value" of the background stress corresponding to the instantaneous value of pulse slip. This allows for a simple prediction that steadily propagating pulses are unstable when the corresponding pulse slip is a decreasing function of the background stress. This is the case for TP pulses (rendering them unstable), and, likely more generally, for fault rheologies characterized by strong weakening with accrued slip and/or slip rate.

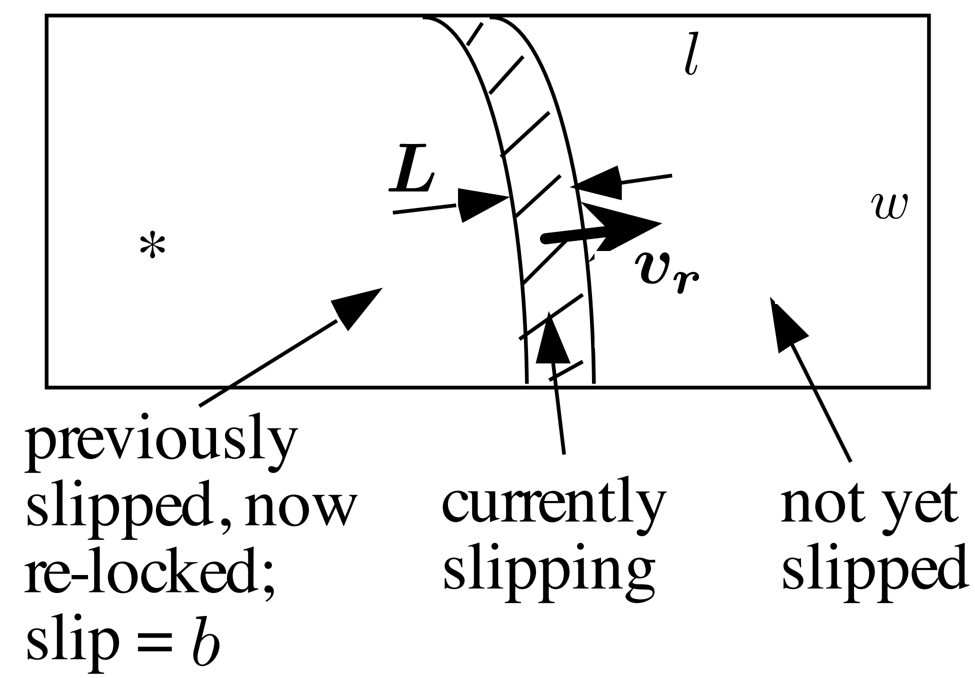
EoM indicates that the pulse perturbation growth is strongly increasing function of the background stress. This leads to steady-state pulses at low background stress to be practically stable, sustained form of rupture (i.e. destabilized at a very slow rate), and only weakly affected by possible fault heterogeneity. Although harder to nucleate (require larger nucleation region), such pulses thus present a candidate rupture mode for large earthquakes on mature (low stress) faults.

Self-Healing Pulse Rupture in Seismological Observations

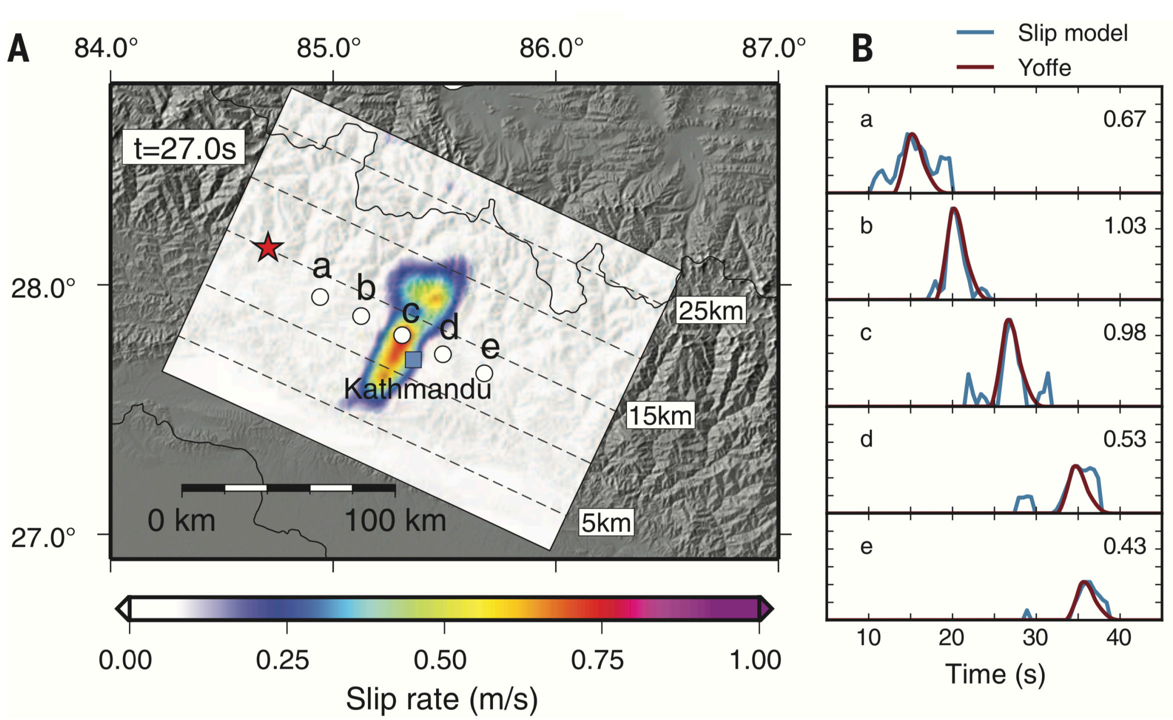
Simplified summary of seismic slip inversion results (following Heaton, 1990):



View onto fault plane during rupture:



Examples of slip pulse kinematic inversions



Slip history from Gorkha M=7.8 Earthquake, Galetzka et al. (2015)

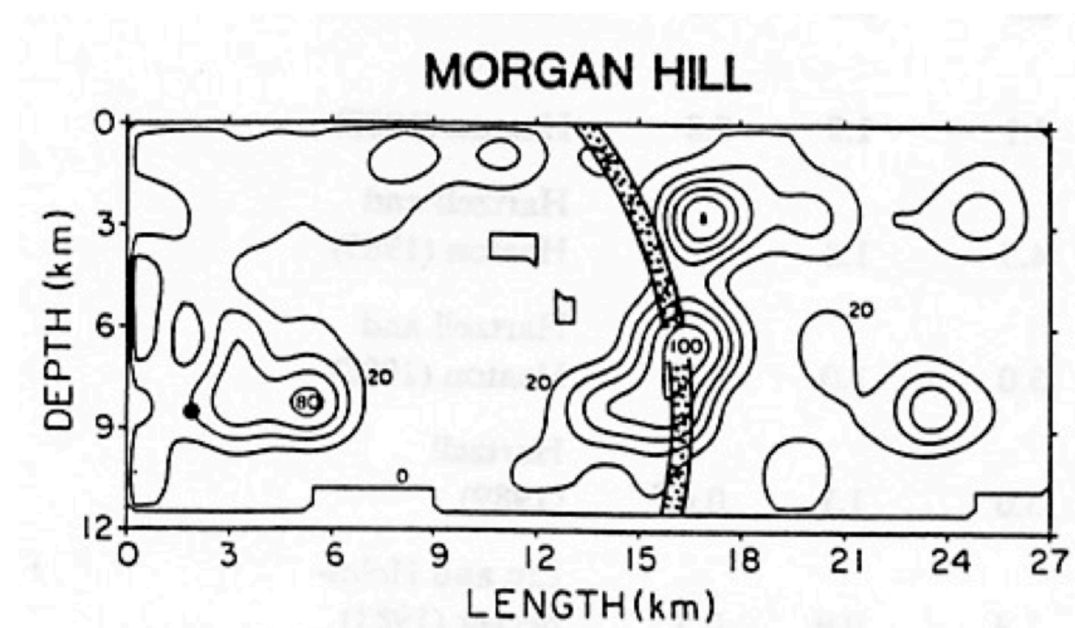


Fig. 5. Slip distribution (cm) for the $M = 6.2$ 1984 Morgan Hill, CA, earthquake derived by Hartzell and Heaton (1986)

Pulse Rupture Source Parameters Compilation

[Garagash, JGR 2012]

Slipped Fault Dimension | Rupture Speed | Pulse Width

Table 1. Earthquake Source Parameters[a]

Event	M_o (10^{18}Nm)	Depth (km)	$\sqrt{S}$[b] (km)	c_s (km/s)	v_r (km/s)	T[c] (s)	L (km)	$\bar{\delta}$ (m)	$\delta_{\max}$ (m)	G[d] (MJ/m^2)	Reference to Kinematic Model[e]
Maule '10	16700	10–65	312	4.7	2.3[f]	18	42	2.4	14	–	A. Sladen and S. Oweng[g]
Nias-Simeulue '05	10000	4–59	275	4.5	2	18	37	2	15	–	*Konca et al.* [2007]
Pisco '07	1210	3–76	99	4.7	2.6[f]	13	33	1.8	–	–	1
Fault 1	364	32–76	63	4.7	2.5[f]	7.9	20	1.3	–	–	
Fault 2	844	8–31	76	4.7	2.8[f]	15	41	2.1	–	–	
Michoacan '85	1500	6–40	155	3.7	2.6	5	13	1.7	6.5	–	2, *Mendoza and Hartzell* [1988]
Benkulu '07	5130	3–98	208	4.7	2.65[f]	6.8	18	1.7	9.6	–	*Konca et al.* [2008]
Pagai Island '07	1100	8–57	130	4.7	2.3[f]	3.3	7.5	0.92	6	–	*Konca et al.* [2008]
Chi-Chi '99	470	0–20	58	3.4	2.5	3.5	8.8	4.5	20	–	*Ma et al.* [2001]
Denali '02	757	0–18	73	3.6	3.3	6.7	22	4.1	11	41.4	*Asano et al.* [2005]
Kashmir '05	282	0–17	50	3.5	2	3.5	7	3.4	10	–	*Avouac et al.* [2006]
Landers'92	77	0–15	33	3.5	2.7	3.5	9.4	2.1	7.9	40.5	*Wald and Heaton* [1994]
Hector Mine '99	63	0–16	32	3.6	1.9	3	5.6	1.8	7	81.2	*Ji et al.* [2002]
Fault 1	26	0–13	21	3.6	1.8	3	5.5	1.7	7	–	
Fault 2	23	0–16	17	3.6	1.8	3.15	5.7	2.3	6.8	–	
Fault 3	14	0–11	17	3.6	2.1	2.5	5.2	1.4	5	–	
San Fernando '71	7	3–16	12	3.5	2.8	0.8	2.2	1.4	2.5	–	2, *Heaton* [1982]
Loma Prieta '89	30	2–20	26	3.6	2.7	1.25	3.2	1.3	4.9	–	*Wald et al.* [1991]
W. Tottori '00	20	1–18	24	3.5	1.8	2.5	4.5	1	2.8	14.3	3
Northridge '94	12	5–20	20	3.6	3	0.92	2.8	0.85	3.2	11.5	*Wald et al.* [1996]
Fukuoka '05	11.5	1–19	22	3.5	2.1	1.9	4	0.76	2.7	10.7	*Asano and Iwata* [2006]
Superst. Hill '87 (3)	3.5	1–12	13	3.2	2.4	0.77	1.8	0.7	1.9	–	*Wald et al.* [1990]
Superst. Hill '87 (2)	0.91	1–12	7.4	3.2	2.4	0.72	1.7	0.6	2.7	–	*Wald et al.* [1990]
Superst. Hill '87 (1)	0.44	1–12	5.9	3.2	2.4	0.71	1.7	0.46	1	–	*Wald et al.* [1990]
Kobe '95	24	0–20	35	3.5	2.8	1.64	4.5	0.63	3.5	3.32	*Wald* [1996]
Borah Peak '83	23	1–21	37	3.5	2.9	0.6	1.7	0.52	1.5	–	2, *Mendoza and Hartzell* [1988]
Imperial Valley '79	5	0–11	19	3.1	2.6	1	2.6	0.5	1.8	3.64	2, *Hartzell and Heaton* [1983]
Colfiorito '97 (Oct)	0.65	3–7	7	3.1	2.3	1	2.3	0.52	0.77	2.22	4
Colfiorito '97 (0940)	1	1–6	9.7	3.1	1.8	1	1.8	0.44	1.4	1.94	4
Colfiorito '97 (0033)	0.44	3–7	6.6	3.1	2.2	1	2.2	0.4	0.64	0.8	4
Morgan Hill '84	2.1	1–12	13	3.1	2.8	0.3	0.84	0.45	1	–	2, *Hartzell and Heaton* [1986]
Morgan Hill '84	2.7	3–13	16	3.5	2.78	0.2	0.56	0.33	2.3	2.72	*Beroza and Spudich* [1988]
Coyote Lake '79	0.35	3–10	6.3	3.3	2.8	0.5	1.4	0.31	1.2	–	2, *Liu and Helmberger* [1983]
N. Palm Springs '86	1.8	4–15	17	3.8	3	0.4	1.2	0.16	0.45	–	2, *Hartzell* [1989]
Parkfield '04	1.1	1–14	19	3.6[h]	3[i]	1.06[i]	3.2	0.09	0.45	0.42	*Custodio et al.* [2009]

[a] M_o, seismic moment; depth range; S, slipped fault area; c_s, shear wave speed in the main source region; v_r, rupture propagation speed; T, duration of slip (rise time); L, length of the slipping patch = $v_r T$; $\bar{\delta}$, average slip in the wake of the pulse = $M_o/\mu S$, assuming $\mu = \rho c_s^2$ and ρ = 2700 kg/m^3 for all but few relatively deeper events (Maule, Nias-Simeulue, Pisco, Benkulu, and Pagai Island, ρ = 3200 kg/m^3); $\delta_{\max}$, maximum slip; G, fracture energy.

[b] S values calculated from the kinematic slip model's data available from the Web sites of M. Mai, D. Wald, and A. Sladen (http://www.seismo.ethz.ch/static/srcmod/, http://earthquake.usgs.gov/regional/sca/slipmodels.php, and http://www.tectonics.caltech.edu/sliphistory/, respectively).

[c] T values are the average slip duration over subfaults with significant slip: the lower significant slip bound is $\bar{\delta}/3$ for all events but the Maule, Nias-Simeulue, and Pisco (2 m); and the Benkulu and Pagai Island (est. 0.75 m based on *Konca et al.* [2008]). For inversions with multiple (≥3) time windows, we partition slip episode(s) on a subfault by discounting time windows with negligible slip (<10% of the maximum window slip), and then choosing the longest continuous slip episode from the remaining time-windows. For the Kashmir event, we use median of the 2–5 s range [*Avouac et al.*, 2006].

[d] G values by *Tinti et al.* [2005, 2008] and *Cocco and Tinti* [2008] (their average breakdown work).

[e] References to kinematic slip models are 1, *Sladen et al.* [2010], model with 38 s time delay of the rupture front between the two fault segments; 2, *Heaton* [1990]; 3, H. Sekiguchi (unpublished manuscript, 2002) as reported by *Tinti et al.* [2005]; 4, preliminary model of *Hernandez et al.* [2004] as reported by *Tinti et al.* [2005]; and other references as noted within Table 1.

[f] Average value of v_r estimated from arrival time contours provided in the references.

Average Characteristics:

- Pulse Width / Fault Dimension

  $L/\sqrt{S} = 0.18 \pm 0.1$

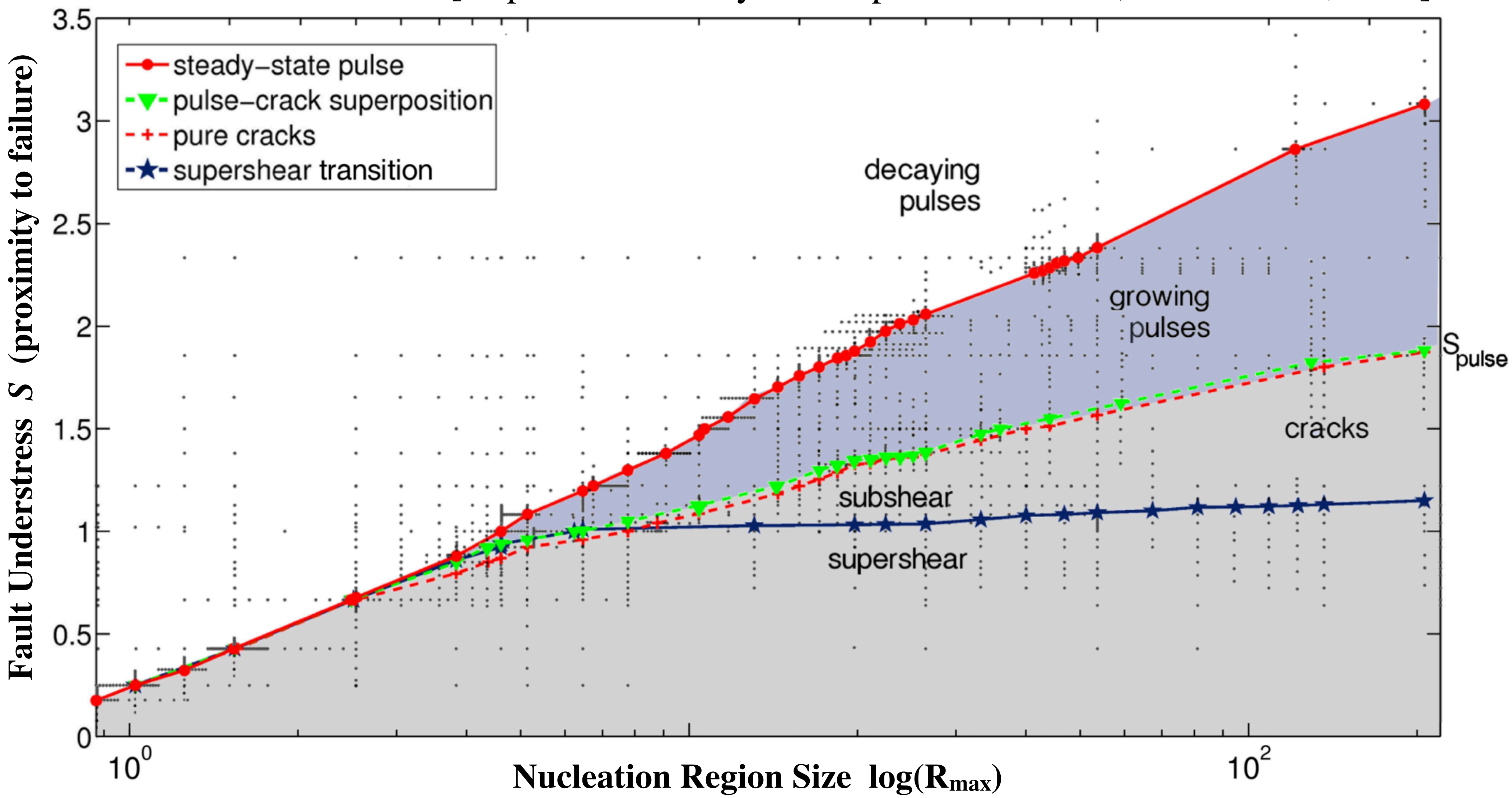
- Rupture Speed / Limiting Speed

  $v_r/c_s = 0.69 \pm 0.14$

Self-Healing Pulse Rupture in Numerical Models

Pulses arise *spontaneously* in dynamic simulations [Gabriel et al. 2012; Noda et al 2009], but **are of *transient* nature - arresting or growing**, and in latter case possibly transitioning to crack-like and/or supershear ruptures. Transient pulse dynamics is a function of the nucleation process (e.g. size of the overstressed nucleation region) and the fault prestress.
Steadily propagating pulses separate the parametric regions of the arresting and the growing pulses, and appear to be unstable.

[Ruptures driven by rate-dependent friction, Gabriel et al, 2012]



Main Questions

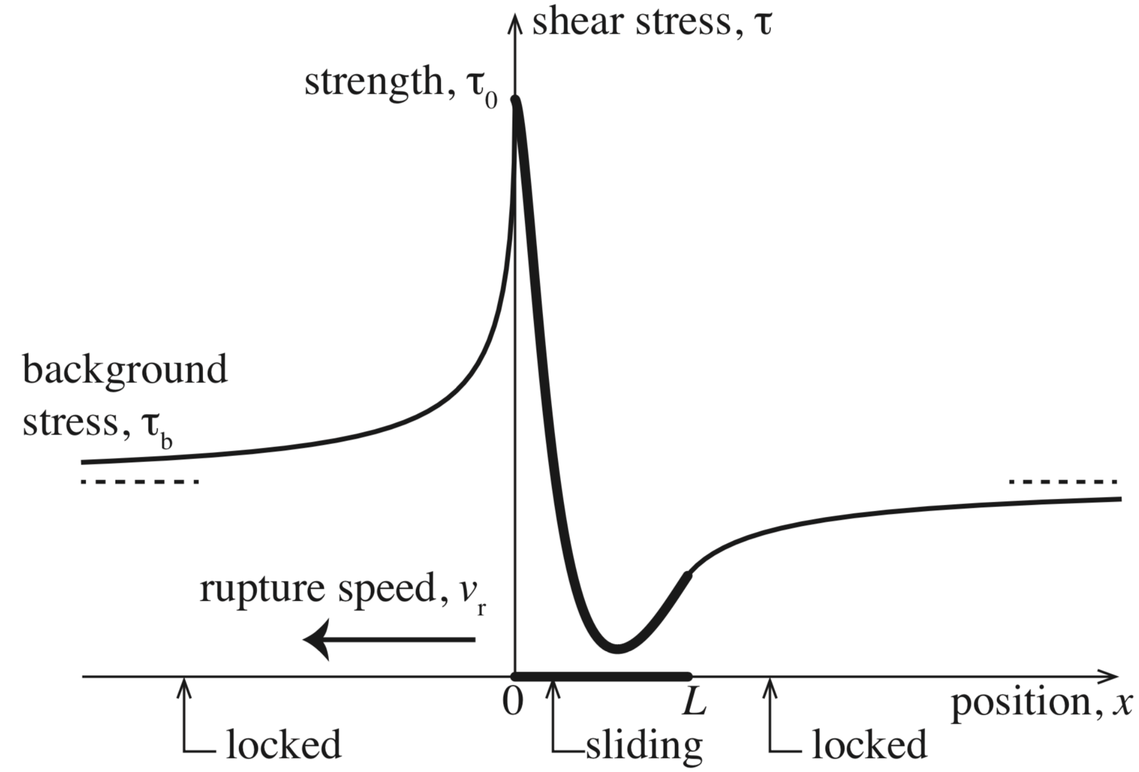
- **What controls stability and dynamics of slip pulses?**
- If modelled pulses are inherently unstable (either quickly arrest or grow out / accelerate to the limiting speed), **how one to reconcile seismological observations of *sustained, sub-limiting speed earthquake pulses ?***

Approach

- Start with "simpler" steadily-propagating pulse solution, which provides bulk pulse characteristics, such as total slip $\boldsymbol{b}$, rupture velocity $\boldsymbol{v_r}$, etc as function of background stress $\boldsymbol{\tau_b}$ and material/fault parameters
- Derive Equation of Motion (EoM) for transient pulse propagation, assuming that instantaneous state of transient pulse can be approximated by a steady-pulse solution at a certain background stress level (the difference between that 'steady-state' background stress level and the actual background stress drives the pulse evolution)
- Validate EoM by full elasto-dynamic numerical solution for a perturbation about a steadily-propagating pulse solution
- Use EoM to quantify the transient pulse dynamics (e.g. growth rate of perturbations from the steady-state pulse solution)

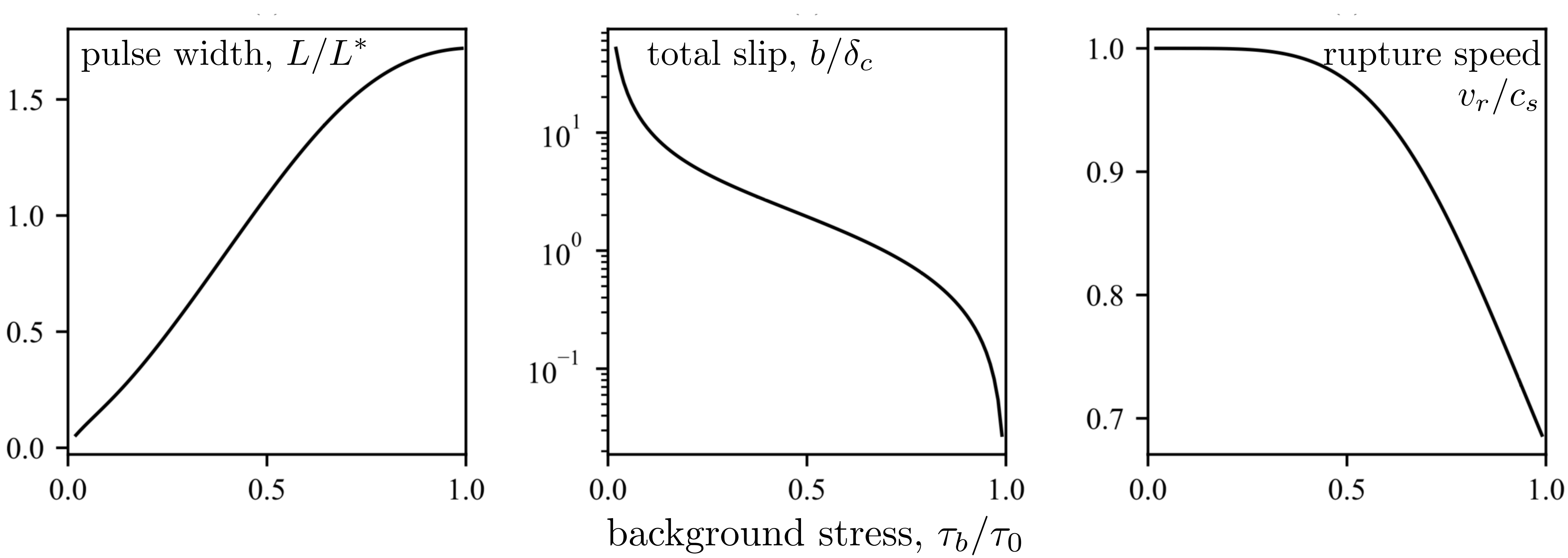
Steady-State Pulses

Steadily-propagating, self-healing pulse solutions are easier to obtain and to fully characterize, e.g., Perrin et al. (1995) and Gabriel et al. (2012) for a fault with rate and state dependent friction, Garagash (2012) for a fault weakened by thermal pressurization (TP), and Platt et al. (2015) for a fault undergoing TP and thermal decomposition



Given material/fault parameters, the pulse characteristics are unique function of the background shear stress: total slip $\boldsymbol{b}(\tau_b)$, pulse width $\boldsymbol{L}(\tau_b)$, rupture velocity $\boldsymbol{v_r}(\tau_b)$, etc

For TP-driven steady-pulses (Garagash 2012)
for the sheared gouge thickness h/h_{dyna}=1 and hydro-to-thermo-diffusivity-ratio α_{hy}/α_{th}=1



Scales:

$$\delta_c = \frac{\rho c}{f\Lambda} h$$

$$L_* = \frac{\mu}{\tau_0} \delta_c$$

$$h_{\text{dyna}} = \frac{L_*}{h} \frac{4\alpha}{c_s}$$

Transient Pulse Equation of Motion (EoM)

At distances x >> pulse width L, transient pulse is seen as a dislocation *b*(*t*) propagating with $v_r(t)$. The corresponding stress field change along the fault can be expanded around that of the steady-state pulse [Weertman 1980] with the instantaneous properties of the transient pulse (φ_{ss}) and the next order correction ($\Delta\varphi$) [Eshelby 1953; Ni and Markenscoff 2009; Brantut et al 2019]

$$\tau(x,t) - \tau_b \approx \phi_{ss}(x; b(t), v_r(t)) + \Delta\phi(x,t)$$

At intermediate distances L_{out} >> x >> pulse width L, where $\boldsymbol{L_{out}}$ is the transitional lengthscale (over which pulse characteristics change appreciably, e.g. $L_{out} \sim b\,/\,(db/dx)$, the pulse stress field is well approximated by the instantaneous steady pulse solution

$$\tau(x,t) - \tau_{b,ss}(b(t)) \approx \phi_{ss}(x; b(t), v_r(t))$$

where $\tau_{b,ss}(b)$ is the steady-state value of the background stress (corresponding to steady pulse with total slip *b*)

Matching the two stress field expansions, and substituting analytical expression for ($\Delta\varphi$) at the intermediate distances from the pulse, we arrive to the **transient pulse EoM**

$$\boxed{\tau_b - \tau_{\rm b,ss}(b) \approx \Delta\phi(x,t) \approx \mu\,\Psi(b)\,\frac{db}{dx}}$$

with $\Psi(b) = \dfrac{1}{2\pi}\dfrac{1}{(1-v_{\rm r}^2/c_{\rm s}^2)^{1/4}}\dfrac{d}{db}\left[\dfrac{b}{(1-v_{\rm r}^2/c_{\rm s}^2)^{1/4}}\right]\ln\left[\dfrac{L_{\rm out}}{L}\right]$

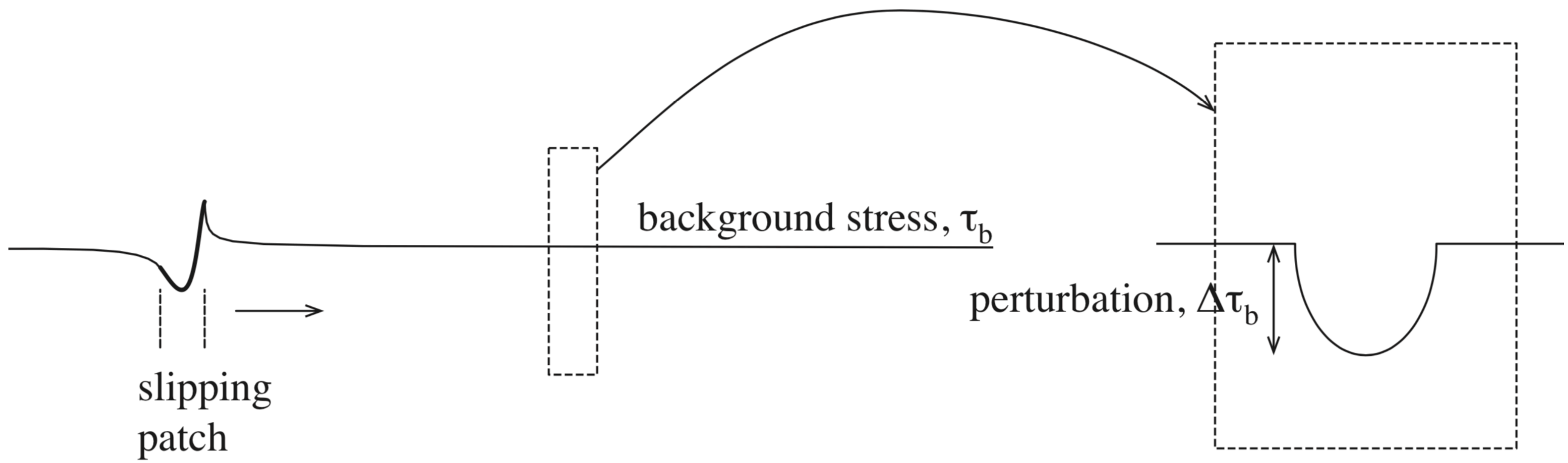
(where $v_{\rm r} = v_{\rm r,ss}(b)$ and $L = L_{\rm ss}(b)$, i.e. steady-state pulse functions of slip *b*)

If $dv_{\rm r,ss}/db > 0$ and $d\tau_{\rm b,ss}/db < 0$, then **steady-state pulses are unstable**
This is the case for pulses driven by thermal pressurization (Garagash 2012) and by rate-weakening friction (Gabriel et al 2012, Perrin et al 1995)

Negative/positive perturbation of τ_b from the steady-state $\tau_{b,ss}(b)$ ➔ ***pulse decay/growth***

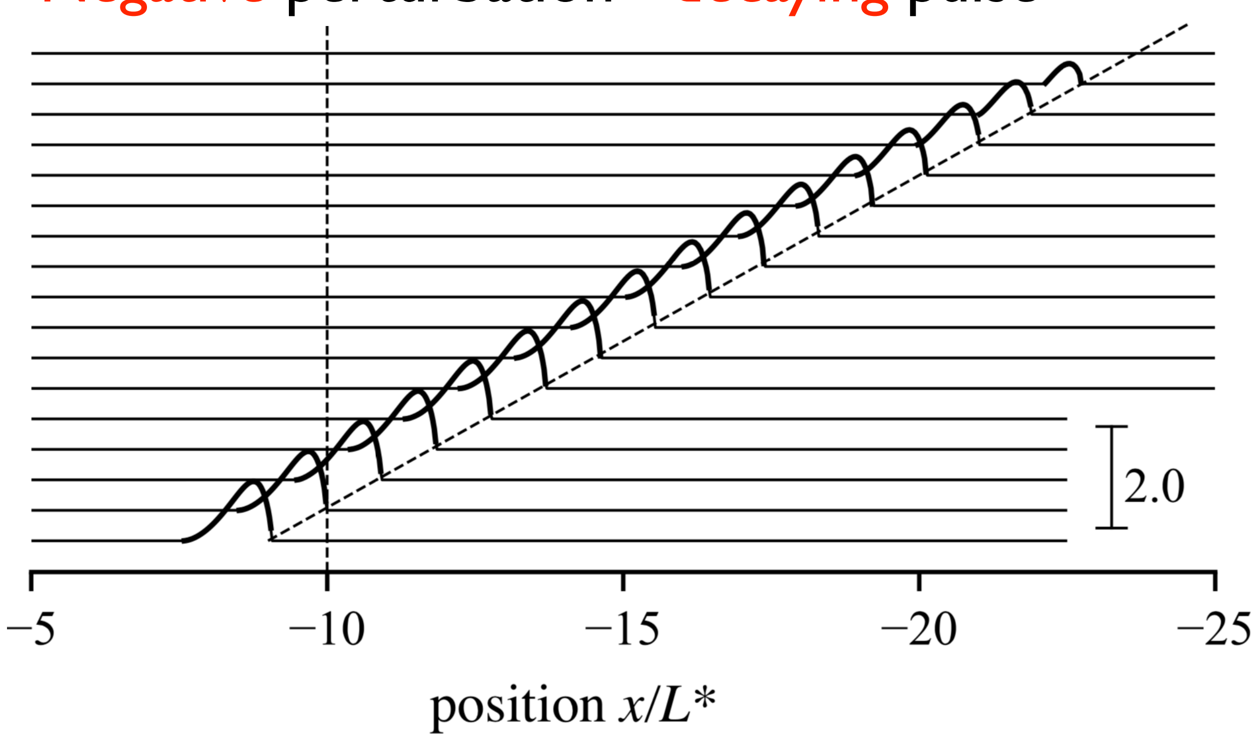
Validation of Pulse EoM

Solve full elastodynamics for perturbations to the steady-state pulse, imposing small variations in background stress ahead of the pulse driven by thermal pressurization.

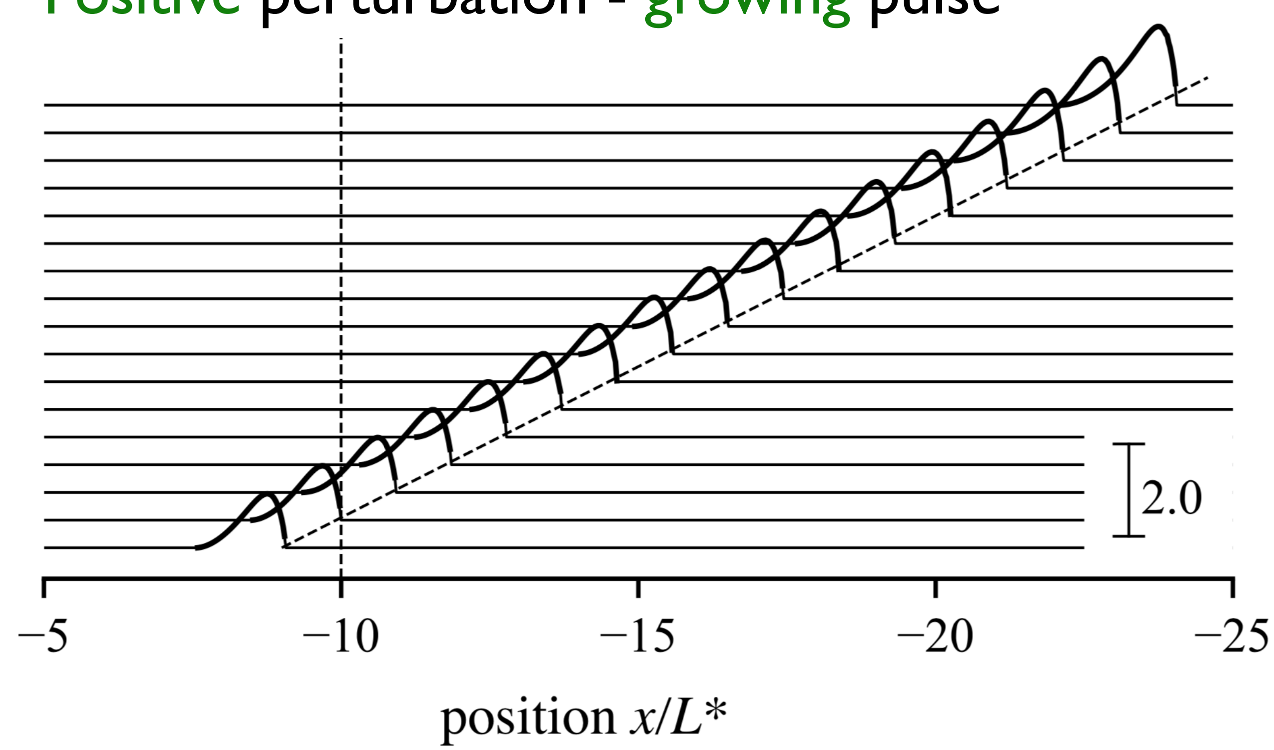


Slip rate V/V* snapshots

Negative perturbation - decaying pulse



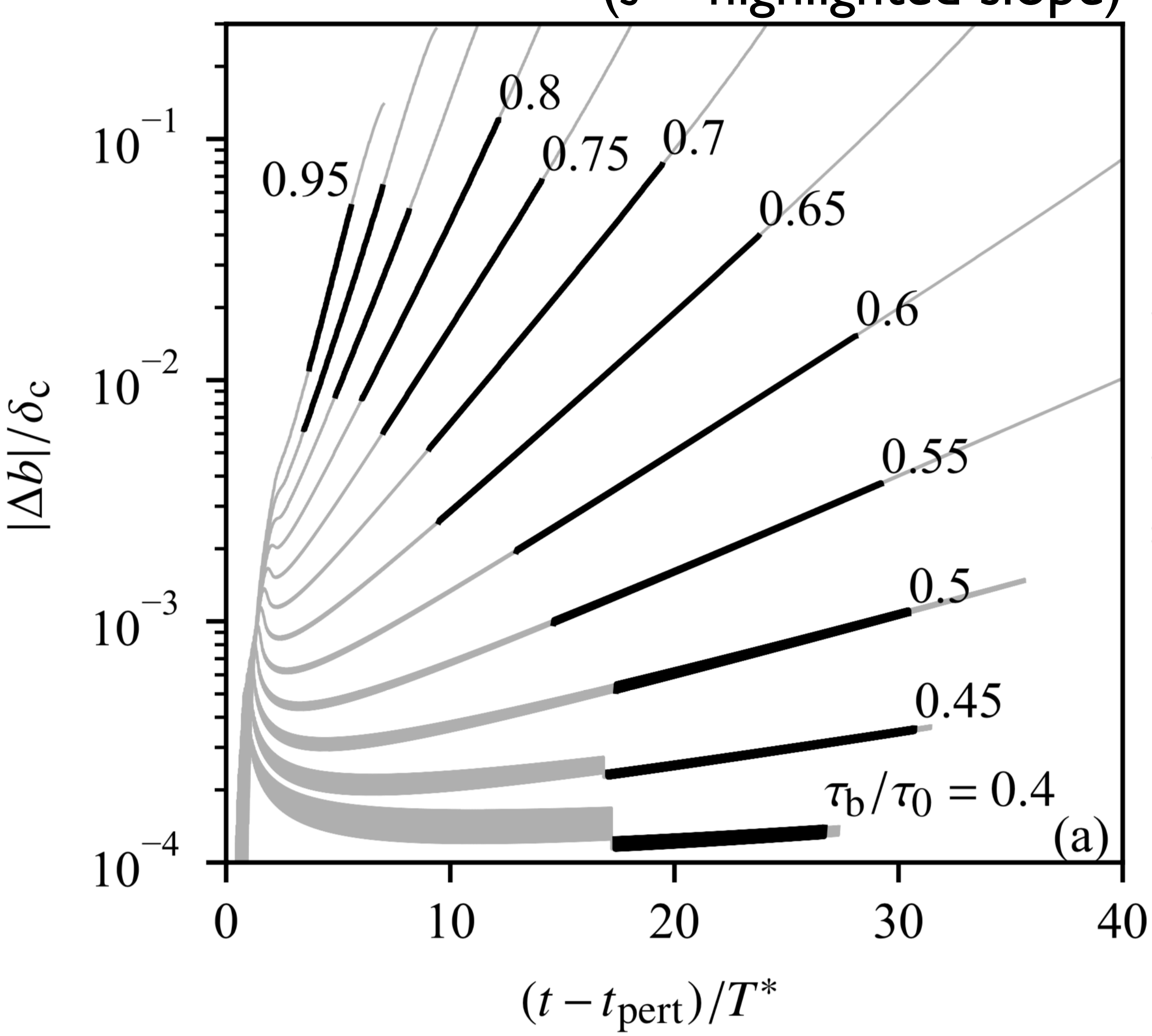
Positive perturbation - growing pulse



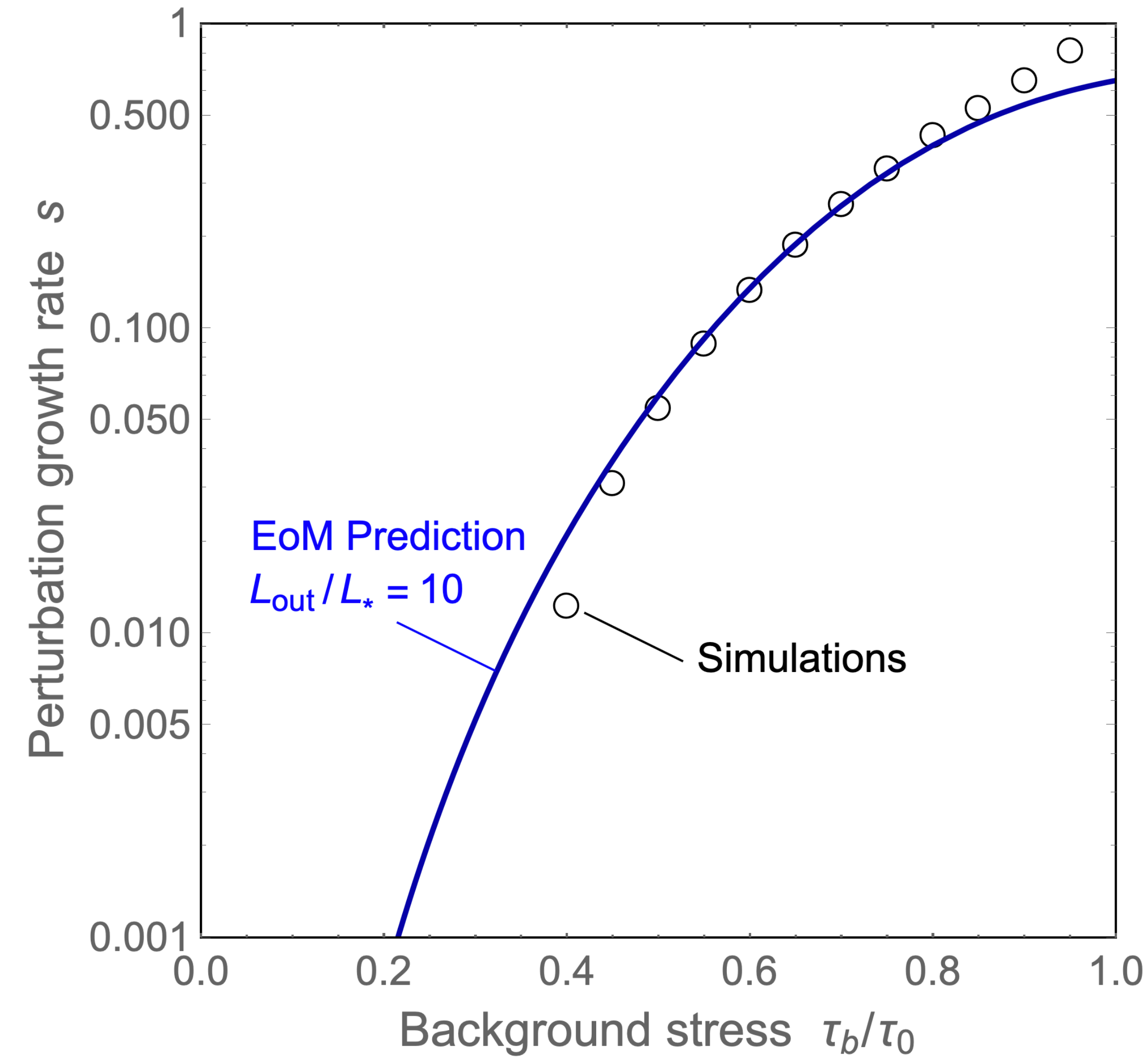
Perturbation growth rate: Elastodynamics Simulations vs. EoM prediction

Growth rate in ***simulations*** $s = \frac{\ln(|\Delta b/\Delta b_{\mathrm{ini}}|)}{\Delta t}$

(s = highlighted slope)



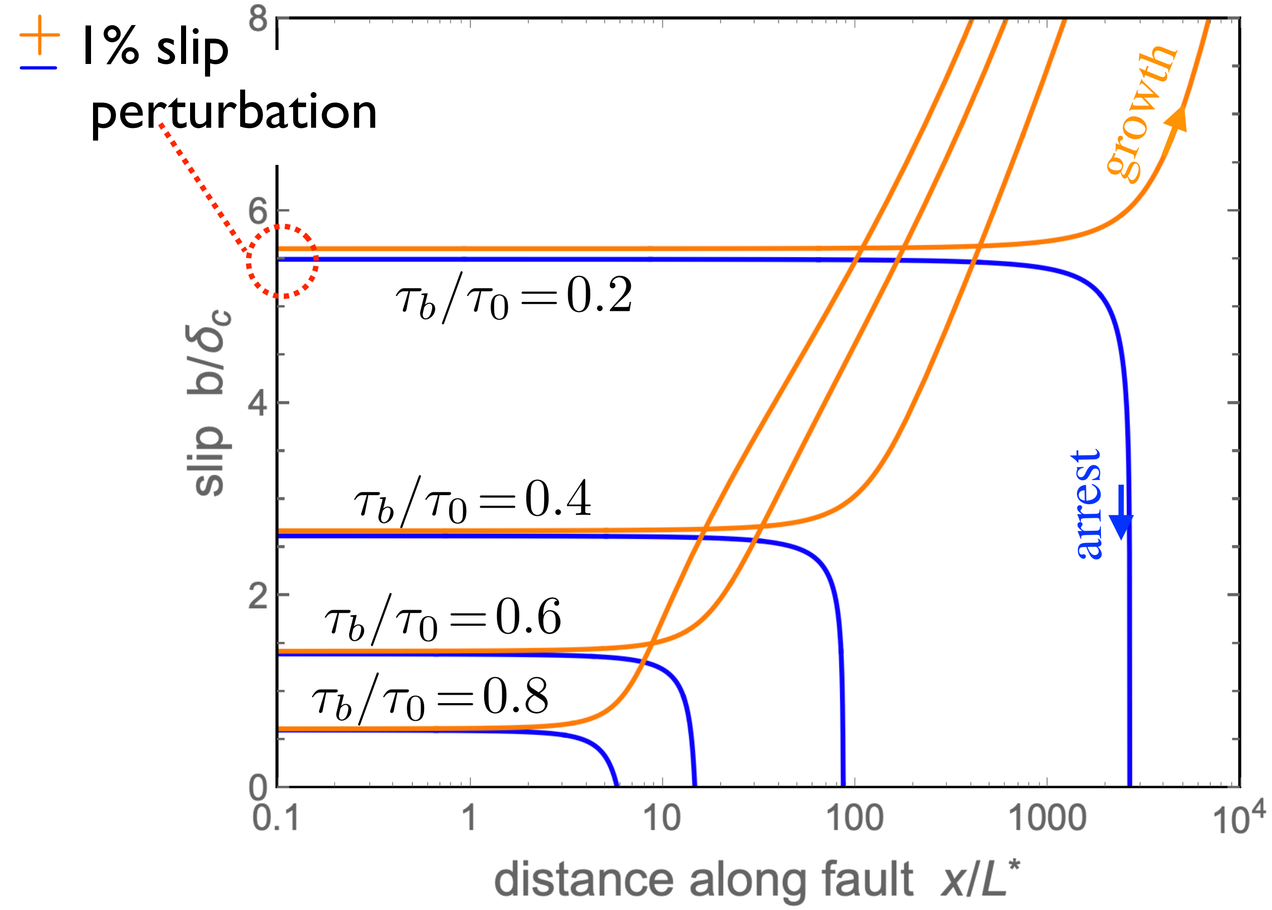
Growth rate from ***EoM*** $s = -\frac{v_r}{\mu\Psi}\frac{d\tau_{\mathrm{b,ss}}}{db}$



Key observations:

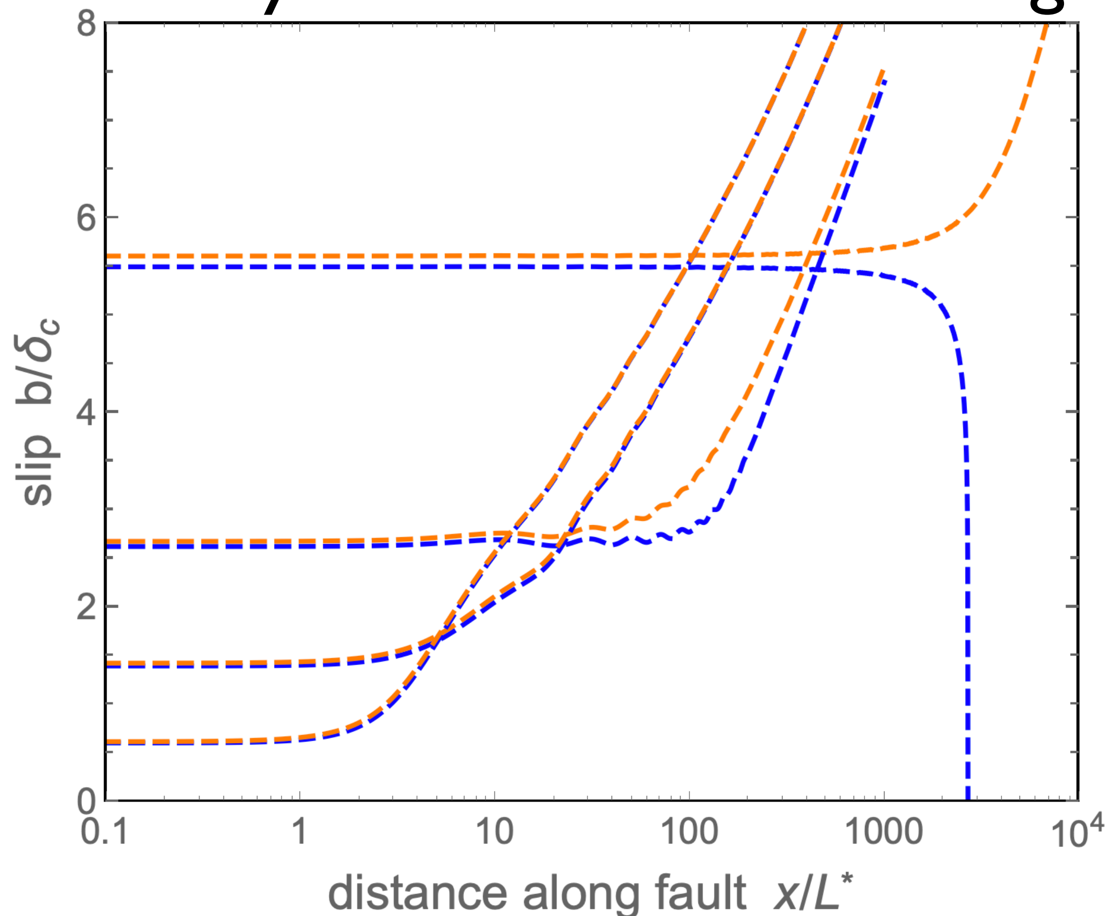
- EoM works (correctly describes pulse perturbation growth)!
- The perturbation growth rate vanishes with background stress ~ $(\tau_b)^5$
  Thus, large-slip (large dynamic weakening) ***steady-state pulses at low background stress are practically stable, sustained form of rupture*** (i.e. destabilized at very slow rate)

EoM Pulse Dynamics: uniform background stress



- Small positive/negative pulse slip perturbation leads to pulse evolution towards eventual growth/arrest
- Pulse evolution at low background stress is very slow, resulting in apparently steady pulse propagation over large distances

EoM Pulse Dynamics: sinusoidal background stress (amplitude 10%, period 20L^*)



- Small (10%) waviness of the background stress does not change the apparent onset of instability
- The sign of the very first stress bump (positive in this case) leads to growth of all pulses (regardless the sign of initial slip perturbation), with exception of the pulse driven by the lowest background stress considered
- The change of sign of the very first stress bump to negative (not shown) would result in all pulses arresting, with exception for the lowest stress pulse

Conclusions

Key points:

- Steady-state slip pulses are unstable if $d\tau_{\mathrm{b,ss}}/db < 0$
- This is the case for thermal pressurization and for rate-weakening friction
- The pulse perturbation growth is strongly increasing function of the background stress

Implications:

- Slip pulses accelerate or stop as the result of very small stress/slip perturbations (no need to have strong asperities/barriers)
- *Larger slip pulses* (undergoing larger degree of dynamic weakening) corresponding to *low level of the fault background stress* are effectively sustained over large run-out distances, and are only weakly sensitive to fault heterogeneities.
  Such pulses, although require larger nucleation region (e.g. Gabriel et al 2012), are a candidate rupture mode for large earthquakes on mature (low stress) faults
- Need for further test the pulse equation of motion on heterogeneous faults

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