Seismic source spectra, stress drop and radiated energy, derived from cohesive-zone models of symmetrical and asymmetrical circular and elliptical ruptures

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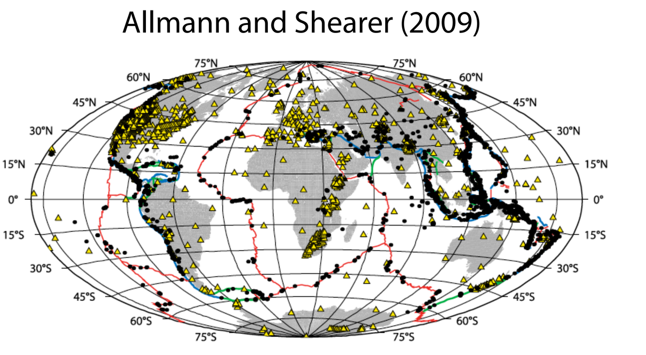
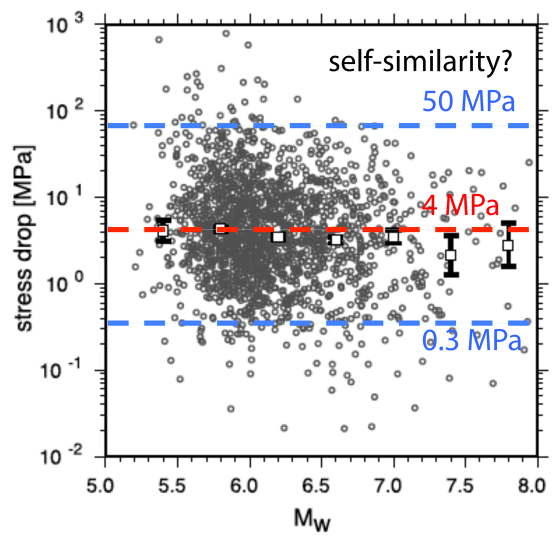
Stress drop of an earthquake estimated from source spectra

Allmann and Shearer (2009)



Analysis of seismic spectra of ~2000 global events



Two issues addressed in this talk:

(1) Absolute value of stress drop estimated from source spectra

(2) Variability of estimated stress drops and radiated energy

Standard method for estimating stress drop from source spectra

Stress drop of a circular fault (Eshelby, 1957)

$$\Delta\sigma = \frac{7}{16}\left(\frac{M_0}{a^3}\right)$$

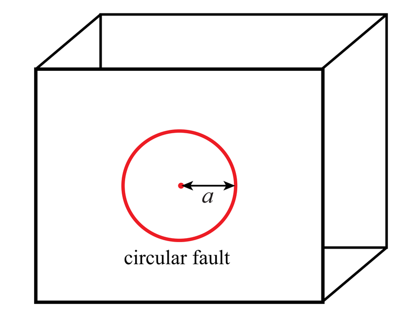
seismic moment; source radius

Corner freq. of spectra

$$f_c = k\frac{\beta}{a}$$

S-wave speed

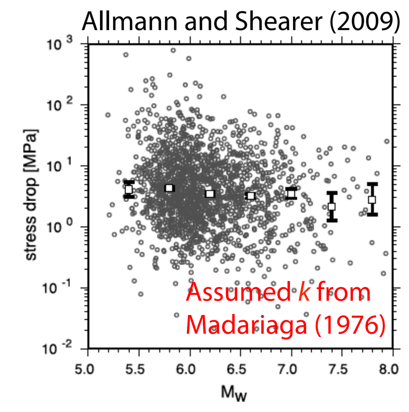
$$\Delta\sigma = \frac{7}{16}\left(\frac{f_c}{k\beta}\right)^3 M_o$$



	k_p	k_s
Brune (1970)	N/A	0.37
Sato & Hirasawa (1973)	0.42	0.29
Madariaga (1976)	0.32	0.21

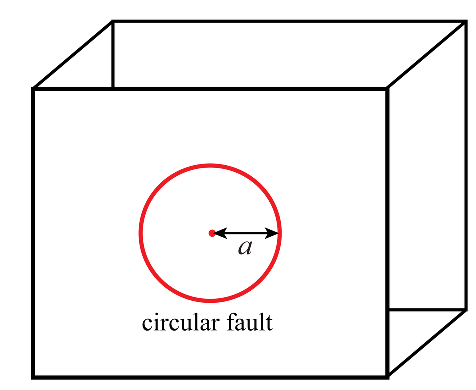
- Brune (1970): A simple, somewhat ad-hoc model of a circular fault, with infinite rupture speed V_r
- Sato & Hirasawa (1973): analytical solution for a singular crack model with instantaneous healing of slip, $V_r = 0.9\beta$
- Madariaga (1976): dynamic calculations for a singular crack with spontaneous healing of slip using FDM, $V_r = 0.9\beta$

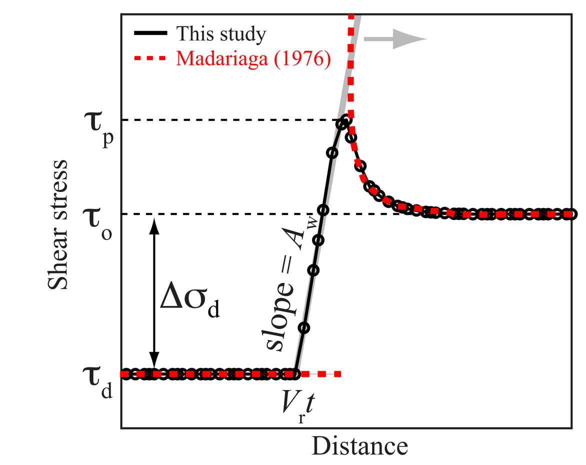
Allmann and Shearer (2009)



A factor of 5.5 difference in $\Delta\sigma$ between Brune and Madariaga

Dynamic model of circular fault: classical problem revisited





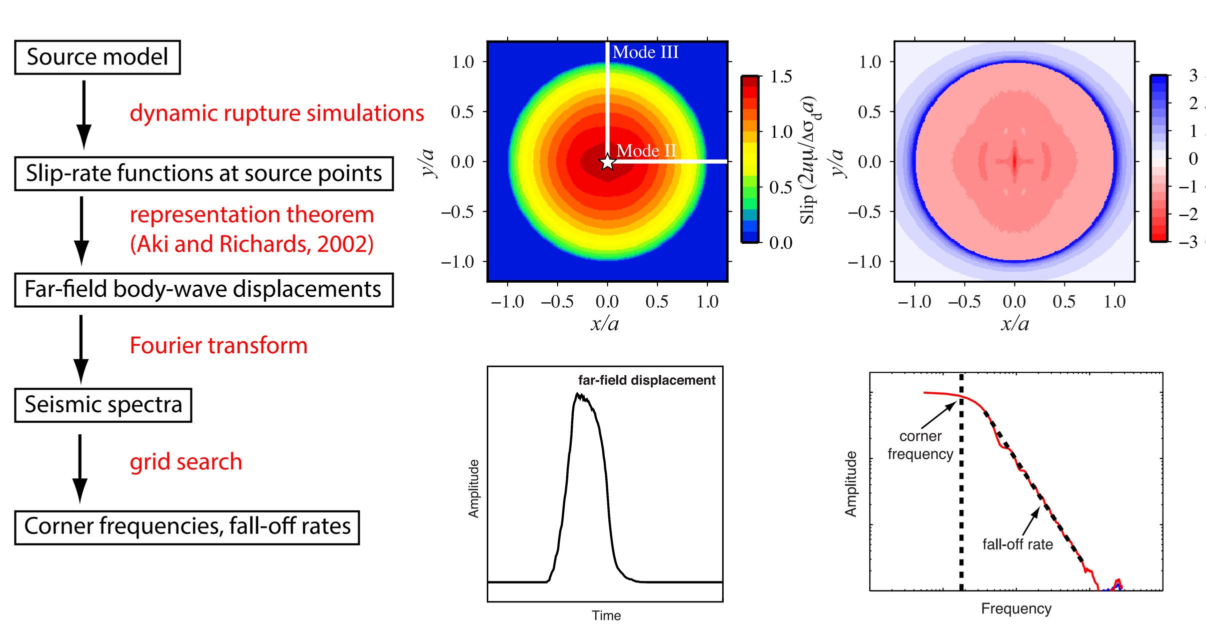
Model of Madariaga (1976):

- Expanding rupture on a circular fault
- Constant rupture speed
- Prescribed dynamic strength drop
- ~~Abrupt change of stress at the rupture front (i.e., singular crack model)~~

$$f_c = k\frac{\beta}{a}$$

Our model: cohesive zone that prevents a stress singularity

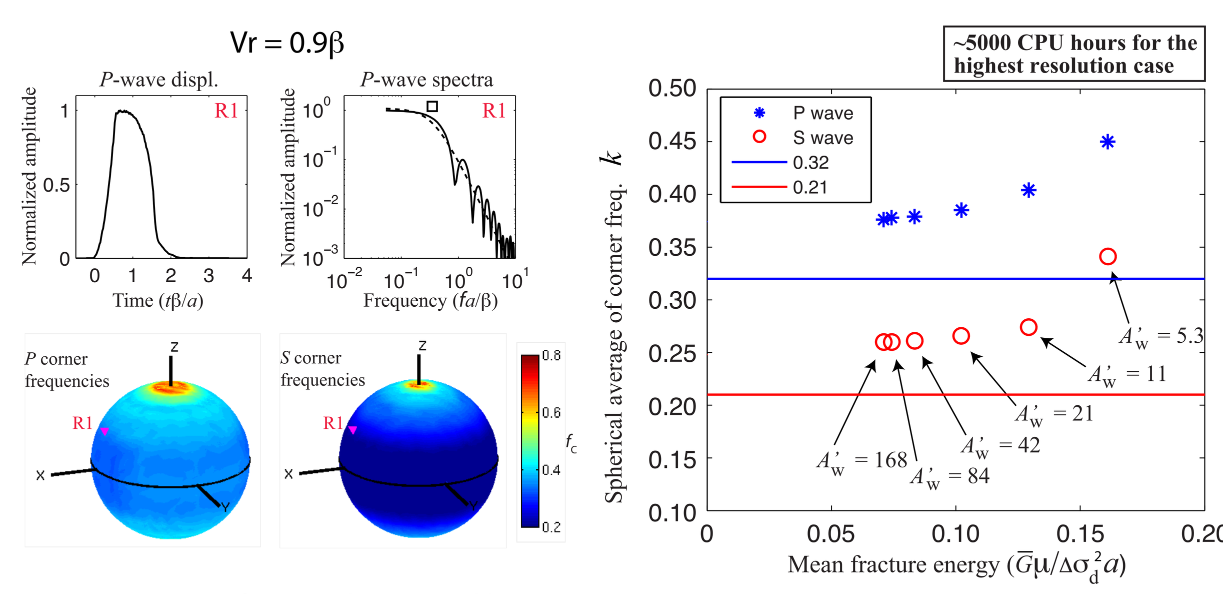
Recipe for computing corner frequencies



$$u(f) = \frac{\Omega_0}{1+(f/f_c)^n}$$

For a model with the smallest cohesive-zone, there are 305 node points along the source radius.

Variation of corner frequencies over the focal sphere

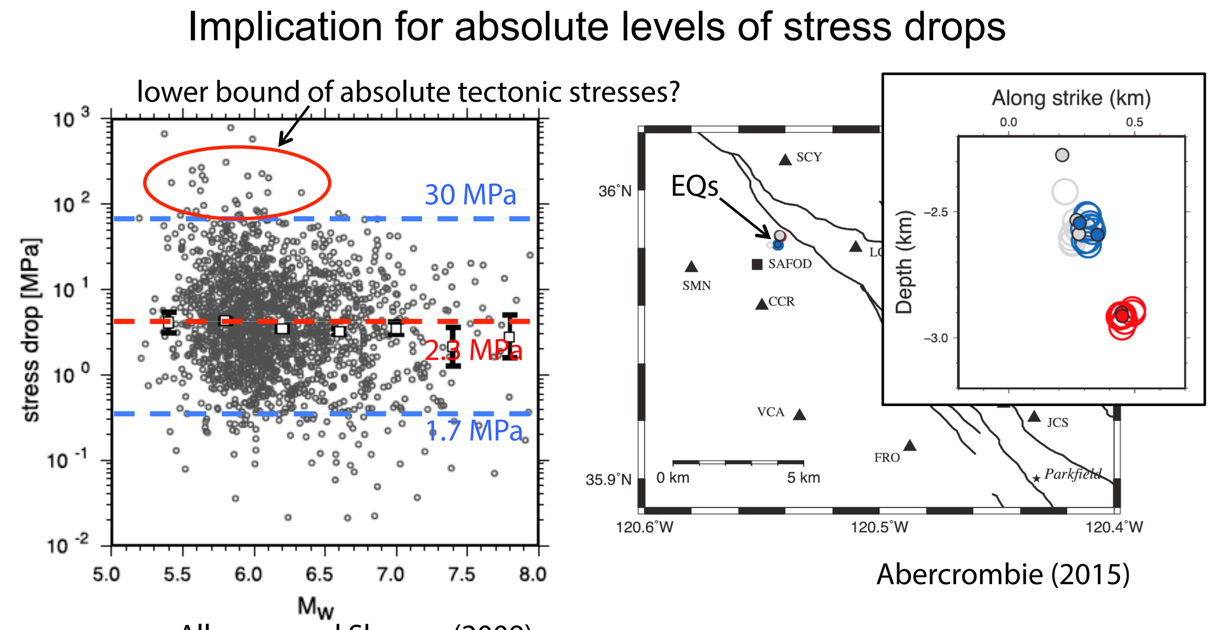


Kaneko and Shearer (GJI, 2014; JGR, 2015)

$$f_c = 0.38\frac{\beta}{a}\ \text{for } P \text{ wave};\quad f_c = 0.26\frac{\beta}{a}\ \text{for } S \text{ wave}$$

- As the weakening rate (A_w) increases, the solution approaches a singular crack model with spontaneous healing of slip and becomes independent of A_w.
- Spherical average of f_c is larger by about 20% than that of Madariaga (1976).
- k is smaller than the model of Sato & Hirasawa (1973) with $k_p = 0.42$ and $k_s = 0.29$.

Implication for absolute levels of stress drops



Allmann and Shearer (2009)

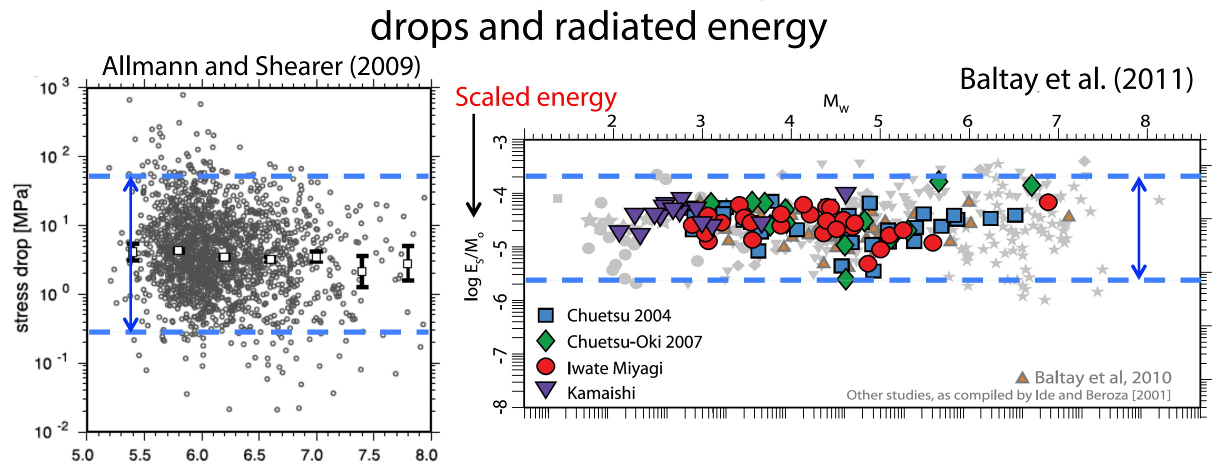
Abercrombie (2015)

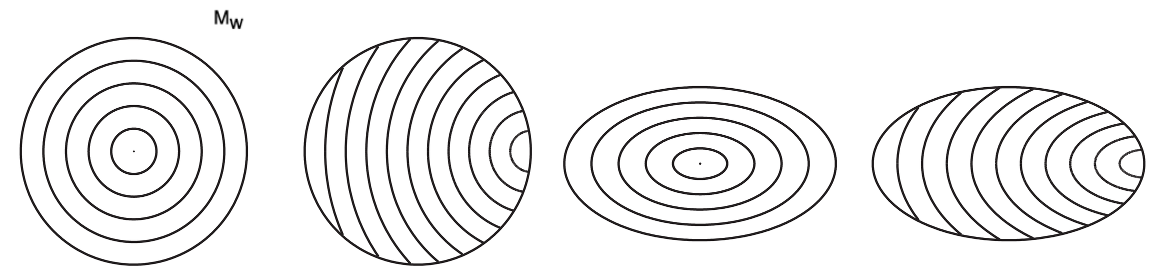
$$\Delta\sigma = \frac{7}{16}\left(\frac{f_c}{k\beta}\right)^3 M_o$$

Application of the Madariaga model overestimates stress drops by a factor of 1.7.

Well-recorded repeating EQs near SAFOD:
$\Delta\sigma$ = 25 – 65 MPa for Madariaga model
$\Delta\sigma$ = 15 – 40 MPa for Kaneko and Shearer (2014) model

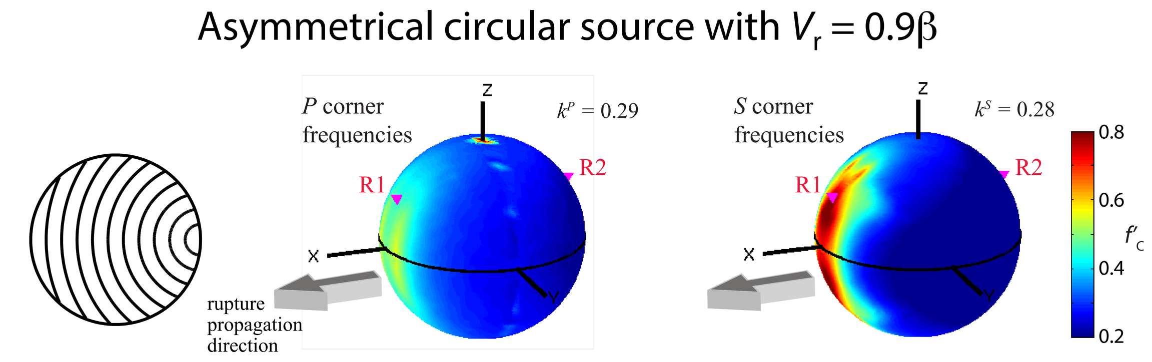
Part 2: Factors contributing to variability of estimated stress drops and radiated energy



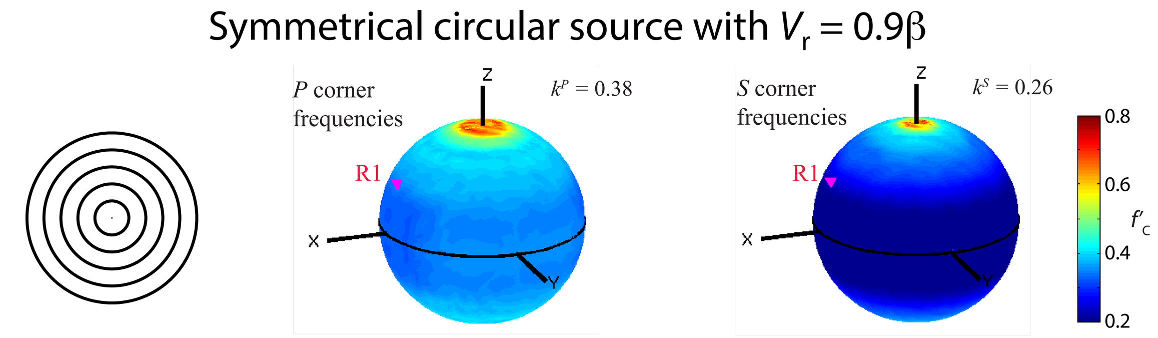


To what extent the variability in seismically estimated stress drops and scaled energy comes from differences in source geometry, rupture directivity and rupture speeds

Asymmetrical circular source with $V_r = 0.9\beta$

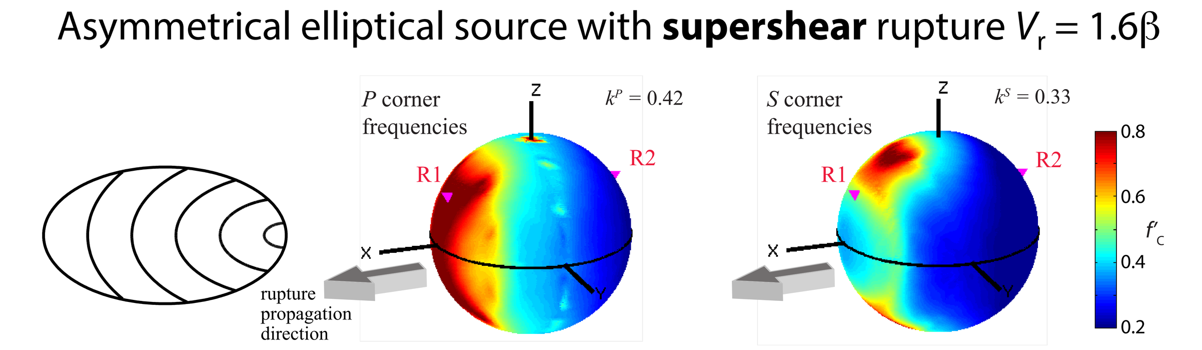


Symmetrical circular source with $V_r = 0.9\beta$

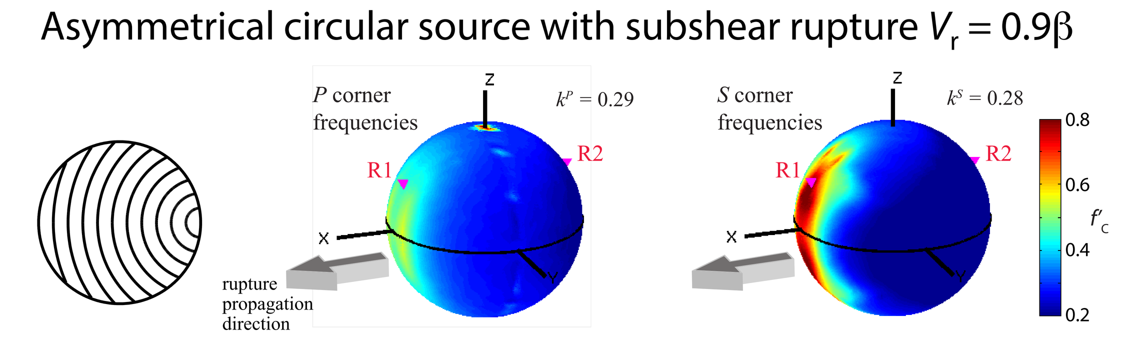


- The asymmetrical model displays a strong azimuthal dependence of f_c due to larger directivity effect
- The spherical average of the S-wave f_c is comparable to that of the symmetrical model

Asymmetrical elliptical source with **supershear** rupture $V_r = 1.6\beta$



Asymmetrical circular source with subshear rupture $V_r = 0.9\beta$

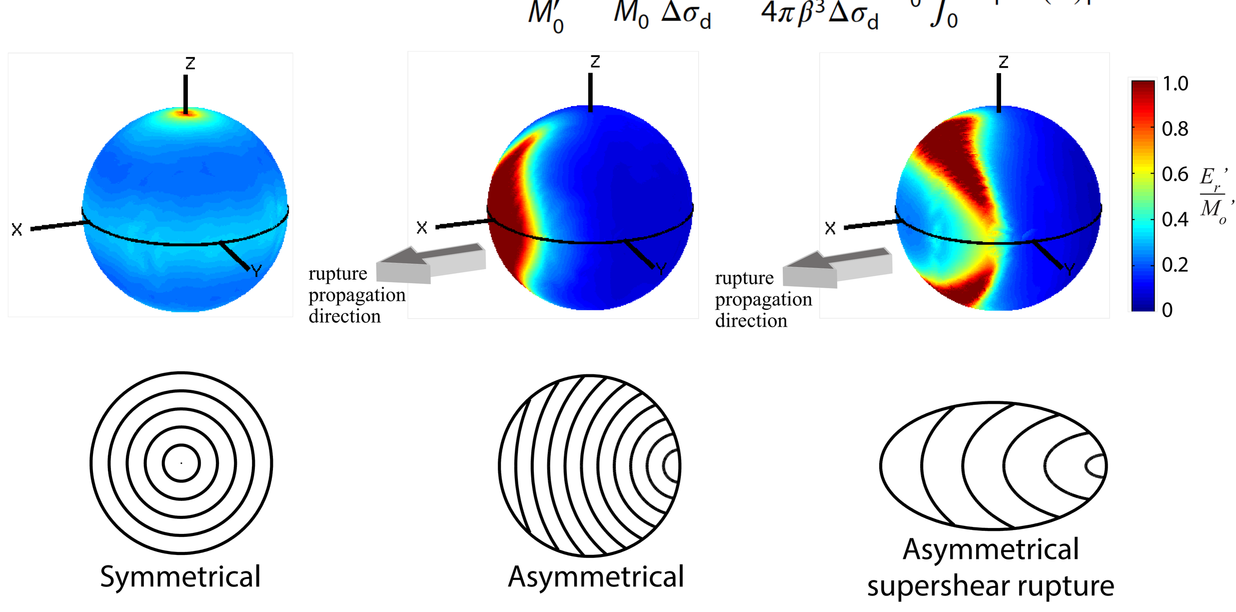


- P-wave f_c are larger than those of subshear rupture
- The pattern of the S-wave f_c is different from that of subshear rupture
- This is caused by Mach waves; the S-wave f_c is largest at the Mach angle $\cos^{-1}(\beta/V_r)$
- A factor of 2 difference is obtained from a variety of source scenarios ($0.6\beta < V_r < 1.6\beta$)

Estimation of scaled energy (E_r^s/M_o)

Non-dimensional scaled energy:

$$\frac{E_r^{\prime s}}{M_0^{\prime}} = \frac{E_r^s}{M_0}\frac{\mu}{\Delta\sigma_d} = \frac{\langle\Psi_S^2\rangle}{4\pi\beta^3\Delta\sigma_d}M_0\int_0^\infty|\omega A(\omega)|^2 d\omega$$



- The asymmetrical circular and elliptical models show large variations in the estimated scaled energy; e.g., for supershear case, E_r^s/M_o ranges from 0.04 to 4.0
- A factor of 5 difference in the spherical average of E_r^s/M_o is obtained from a variety of source scenarios ($0.6\beta < V_r < 1.6\beta$)

Conclusions

- We have re-visited the classical problem of a circular fault and derived a new relation between a source dimension and the spherical average of corner frequencies of far-field body wave spectra.
- In observational studies that assumed Madariaga (1976), the mean value of $\Delta\sigma$ may have been overestimated by a factor of 1.7.
- At least a factor of 2 difference in the spherical average of f_c is expected in observational studies simply from variability in source geometry, rupture directivity, and rupture speeds, translating into a factor of 8 difference in estimated $\Delta\sigma$.
- At least a factor of 5 difference in scaled energy is expected from the variability in the same source characteristics. These numbers increase with an insufficient station coverage (not discussed in this talk).
- Mach waves generated by supershear rupture lead to much higher f_c and scaled-energy estimates locally, suggesting that supershear earthquakes can be identified from the analysis of f_c and scaled energy.

Kaneko and Shearer (GJI, 2014; JGR, 2015)

Future work: Consider rupture characterized by self-healing pulse (e.g., $V_r = 0.9\beta$ and $V_h = 0.7\beta$)