## Radiated energy of shallow moderate-to-large earthquakes

Marine A. Denolle and Peter M. Shearer

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Cecil H. and Ida M. Green





Radiated energy is estimated from the kinetic energy carried by seismic waves

$$E_R = \iint_{S_0} \int_{-\infty}^{\infty} \rho c \, v^2(t) dt dS$$

Most of the energy is carried by high frequencies (mostly P and S)

 $S_0$  is a sphere in the far-field of the source within which we assume a homogeneous whole space

Need to remove wave propagation effects



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#### **Global Green's functions to calibrate path effects**

- 1. Green's function from <u>simulation</u> of wave propagation in 3D Earth
- + would be ideal

### + receivers can be from anywhere

computationally challenging for high frequency Gfs
require accurate high resolution global velocity models

- 2. Empirical Green's function from <u>small</u> events (eGf)
  - + true 3D path
    + aftershocks are great
    candidates for eGf
    + receivers can be from

#### + receivers can be from anywhere

- Small events have lower SNR at long periods
- require knowledge of eGf
   source spectrum

 Attenuation Q and ray tracing in simple PREM/IASPEI 3D Earth

# + computationally inexpensive + good approximation at long period

only valid for direct
waves such that receivers
are restricted to 30-90°
angular distance
does not include lateral
path effects (often called
1D Green's function)

#### Free surface affects the body-wave GF



#### **P** wave train of shallow earthquakes: gP = P + pP + sP



#### **Depth-phase interference**

Boatwright and Choy (1986) propose a depth-phase correction and assume that we can sum the energies:  $E_{gP} = E_P + E_{sP} + E_{pP}$ 



#### Two approaches to estimating radiated energy

2.

Energy flux from Boatwright's 1. work

 $E_B = 32\pi^3 \rho \alpha \int_0^\infty \left| f S(f) \right|^2 df$ Respect use impedance

of Parseval's theorem

velocity<sup>2</sup>

Energy from spectral shape (single-station)  $E_s = \frac{2\pi M_0^2}{2\pi M_0^2} \langle \mathbf{r} \rangle$  $\boldsymbol{J}_0 \mid \boldsymbol{J}$ Independent Velocity<sup>2</sup>: Radiation measure of pattern squared S(f) is seismic averaged over moment the focal sphere to low frequency

normalized asymptote

#### Two approaches to estimating radiated energy

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 Energy flux from Boatwright's work

$$E_B = 32\pi^3 \rho \alpha \int_0^\infty \left| f S(f) \right|^2 df$$

#### + true measure of energy flux

- individual measurements are sensitive to:

- radiation pattern
- geometrical spreading
   require <u>well distributed</u> and <u>dense</u> receivers to capture the entire focal sphere

Often used with "1D" Green's function  $\left|G(f)\right| = \frac{1}{\Re_{P}} \exp(-\pi f t^{*}(f))$ 

Depth-phase correction  $E_{B} = E_{B} / (1 + E_{sP} / E_{P} + E_{pP} / E_{P})$  2. Energy from spectral shape (single-station)

$$E_{S} = \frac{2\pi M_{0}^{2} \langle R_{P} \rangle^{2}}{\rho \alpha^{5}} \int_{0}^{\infty} \left| f \,\overline{S}(f) \right|^{2} df$$

+only relies on spectral shape
+one station is sufficient to
estimate energy
- assumes that the spectral
shape is <u>constant</u> at all azimuth
and takeoff angles
- <u>normalization</u> to low
frequency asymptote sensitive
to SNR and data processing
- assumes a <u>whole space</u>
around the source (not valid for
"very" shallow earthquakes)

Often used with empirical Green's function

#### Two approaches sensitive to depth phases

 Energy flux from Boatwright's work

Tendency to underestimate energy Destructive interferences 2. Energy from spectral shape (single-station)

Tendency to overestimate energy

High apparent corner frequency *if normalization is performed on the low frequency asymptote* 

Correction for both approaches when we use 1D Green's functions
Characterize uncertainties when we use empirical Green's functions

#### **Example : Nepal 2015 earthquake sequence**



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#### Along-dip Main Himalayan Thrust (MHT) geometry









#### Fit source depth



CMT depth: 17.4 km PDE/NEIC depth: 24.8 km

CMT depth: 15 km PDE/NEIC depth: 12 km 

#### Fit source depth



#### Fit source spectrum: corner frequency (ignore depth phase)



#### Fit source spectrum: corner frequency (account for depth phases)





#### **Radiated Energy Correction for depth phase interference**

1) Measure apparent energy from synthetics at each station *i* between  $f_1$  and  $f_2$ :

$$E_{syn}(i, [f_1, f_2])$$

2) Measure total synthetic energy between 0 and  $\infty$ :

$$E_{true}([0, \infty])$$

3) Multiply individual apparent energy measurements with the ratio (2) by (1).



• P wave trains at teleseismic distances from M6+ in the upper 35 km will show interferences of direct and depth phases.

• Interferences significantly bias estimates of corner frequencies and low frequency asymptote.

• Bias in radiated energy estimates is controlled by source depth and source size (~ corner frequency).





#### Estimates from finite sources: low and high frequency backprojection

From Wenyuan Fan and Peter Shearer (Fan and Shearer, accepted in GRL)









**Estimates from finite sources: Stage #1** 





#### From stage #1 to stage #2: passage through a lateral ramp

J.L. Mugnier et al. / Tectonophysics 509 (2011) 33-49



#### From stage #1 to stage #2: passage through a lateral ramp





![](_page_37_Figure_0.jpeg)

#### **Conclusion on the beginning of the Nepal M7.8 earthquake**

![](_page_38_Figure_1.jpeg)

#### **Concluding remarks**

• If not properly removed from path effects, direct and depth phases interference greatly affects the P spectral shape.

• Interferences introduce large biases in corner frequency and low-frequency asymptote.

• We can properly estimate the source depth, spectra, and radiated energy by best fitting the predicted P wave train spectral shape.

• We applied these methods to the Nepal earthquake sequence.

• We expanded the point source approximation to look into finite source effects on high frequency radiated energy.

#### Questions

• Are depth-phase interference responsible to the apparent scaling in scaled energy  $E_R / M_0$  (in cases of shallow large seismicity)?

• With refined depth and radiated energy estimates, do we see trends radiated energy with depth?

• When earthquake size > earthquake depth, the homogeneous whole space assumption fails. What do we do?