Uncertainty Quantification and Quality Appraisal for Finite-Fault Earthquake Source Inversions



SUMMARY

Finite-fault source inversions estimate kinematic rupture parameters of earthquakes using a variety of available data sets and inversion approaches. Rupture models are obtained by solving an inherently illposed inverse problem, subject to numerous a priori assumptions and noisy observations. Despite these limitations, near real-time source inversions are becoming increasingly popular, while we still face the dilemma that uncertainties in source inversions are essentially unknown. Yet, the accurate estimation of earthquake rupture properties, including proper uncertainty quantification, is critical for earthquake seismology and seismic hazard analysis, as they help to characterize earthquake complexity across all scales.

The "Source Inversion Validation" (SIV) project is a collaborative international multi-institutional effort to examine current state-of-the-art in earthquake source inversion, and to develop and test novel source inversion approaches. Through a series of benchmark exercises of varying degree of complexity, we test inversion methods, and evaluate their performance through different comparative metrics. We quantify the intra-event variability in rupture models, which is evident for past earthquakes in the SRCMOD database (<u>http://equake-rc.info/srcmod</u>), and propose metrics to rank earthquake rupture models.

Here, we summarize the SIV-efforts and latest results, and describe several quantitative metrics to parameterize the similarity (or dissimilarity) of kinematic source properties (e.g. slip on the fault) that help to assess the quality and model robustness of finite-fault source models.

APPROACH

- Series of benchmarks with varying degree of complexity, with and without "noise" in the data and various levels of information on meta data (e.g. fault geometry, velocity-density structure)
- All benchmarks remain accessible for all interested users; only for the most recent test the solution (input model) is not released
- Use various statistical metrics to quantitatively compare and "rank" models
- For the initial Green's function testing and the first benchmarks, Earth structure and the source-station geometry remains fixed to simplify the work

Sequence of Benchmark Exercises

60

- Point-source forward-calculations for Green's function testing
- Forward-modeling cases for two "simple" extended-fault kinematic cases
- Inv1: Inversion for "simple" M 6.5 strike-slip dynamic rupture model
- Inv2: Inversion for kinematic M 7 normalfaulting scenario, incl. uncertainties in the Green's functions (through 3D scattering)
- Inv3: teleseismic case for very large strike-slip rupture in Southern California
- Inv4: Complex-geometry blind-thrust earthquake in Southern California

P. Martin Mai & many, many SIV-participants*

martin.mai@kaust.edu.sa



INITIAL RESULTS & STATISTICAL METRICS

- INV 1: M 6.5 strike-slip scenario Input for spontaneous dynamic rupture model using heterogeneous initial stress, to generate rupture model and near-fault synthetics for source-site geometry and velocity-density structure (shown on previous panel)
- Kinematic source properties of slip-rate (top left) and final slip (bottom left), and resulting source-time functions at selected points on the rupture plane (bottom right)
- Despite noise-free synthetic data and perfectly known meta-data (Earth structure; geometry), the inversion solution show substantial variations (below: four example solutions from a set of 11 submitted solutions)



Statistical Metrics to Quantify Source-Model Differences

- Waveform misfits are computed using standard norms (L1, L2, variance reduction, cross-correlation)
- . We also report time-frequency envelope (TFE) misfits and time-frequency phase (TFP) misfit to determine goodness-of-fit criteria (Kristekova et al., 2006, 2009)
- 3. Differences in the slip distributions are assessed using the **Spatial Prediction Comparison Test** (SPCT; Hering & Genton, 2011; Zhang et al. 2015) and multi-dimensional scaling (Razafindrakoto et al. 2015) to rank the models
- The statistical tools are developed, tested, and calibrated on a range of synthetic cases, and can be used in case a known solution exist.

 \rightarrow These statistical tools can be applied also to slip models of past earthquakes (with unknown true solution) by defining an appropriate test or "mean" distribution

Brief summary of notation

- $\{Z(\mathbf{s})\}$ - Spatial process Z(s) at locations s
- General loss function for prediction
- and realization
- $g[Z(\mathbf{s}_i), \hat{Z}_P]$ - Squared-error loss (SE)
- $g[Z(\mathbf{s}_i), \hat{Z}]$ - Absolute-error (AE)
- -Correlation skill (CS) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = \frac{n}{(n-1)}$

http://equake-rc.info/siv

$$g \in \mathbb{R} : \mathbf{s} \in D \subset \mathbb{R}^{2}$$

$$g[Z(\mathbf{s}_{i}), \hat{Z}_{P}(\mathbf{s}_{i})]$$

$$(\mathbf{s}_{i})] = [Z(\mathbf{s}_{i}) - \hat{Z}_{P}(\mathbf{s}_{i})]^{2}$$

$$\hat{\sigma}_{Z} \hat{\sigma}_{P} [Z(\mathbf{s}_{i}) - \overline{Z}][\hat{Z}_{P}(\mathbf{s}_{i}) - \overline{Z}]$$



Slip-inversion results for INV1 benchmark. Top panel shows slip on the fault; bottom *left: rupture time; bottom right: rise time*

₽1 📋

P5 13 112 111 117 114 117 20 P8 19 19 118 P7 114 123 197 FID 125 125 124 121 129 129 FID 125 125 124 121

-40 -50 -40 -30 -20 -10 0 10 20 30 40 Fault-Parallel Distance (km)

xample waveforms misfits at

sed in the inversion

Goodness of fit in envelope

0.0 0 5 10 15 20 25 30

Goodness

es for the inversion (red) and

0.0 0 5 10 15 20 25 30

Synthetic test models,

with identical random

relation lengths

Top (SE, AE): Negative

values (blue) indicate

that the case named

in the corresponding

Bottom (CS): values

(red) indicate that the

case named in the

better based

corresponding row is

row is the better

seed but different

s of fit in phase

Below: the reference solution



	0.010 0.005 0.000 0.010 0.	0.006 + 100 + 10
0020 0015 0000 0000 0000 0000 0000 0000	$\begin{array}{c} 0.03\\ 0.02\\ 0.02\\ 0.00\\$	003 000 000 000 000 000 000 000 000 000
0.10 0.00 0.00 0.00 0.00 0.00 0.00 0.00	$\begin{array}{c} 0.08\\ 0.06\\ 0.02\\ 0.02\\ 0.02\\ 0.02\\ 0.04\\ 0.06\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\$	000 000 000 000 000 000 000 000
0.00 0.00	010 000 -000 -015 -0	0.03 0.02 0.01 0.01 0.01 0.01 0.02 0.01 0.02 0.01 0.02 0.01 0.02
0.00 + 0.000	$\frac{0.00}{0.00} \xrightarrow{\text{invl 052 y}}_{0.00}$	0.03 - (mv1.052 z) 0.03 - (mv1.052 z) 0.03 - (mv1.052 z) 0.03 - (mv1.052 z) 0.03 - (mv1.053 z)

Time-frequency envelope misfit









- how well a given 2D field (slip model) "fits" a reference solution • Using a multidimensional scaling approach allows to further quantify in which sense the models are different (amplitude; patch location ..), and to propose some form of ranking for the models)



CURRENT & FORTHCOMING BENCHMARKS

Multi-dimensional scaling

м	o (Nm)	Mw	Max Slin	Dimensions (km)		Eff Dimensions (km)		Slir	n Centroid	(km)
	(((iii))	14144	(m)	Width	Length	Width	Length	Xc	Yc	Zc
9	.740e+18	6.63	1.3300	19.98	35.00	12.83	23.96	0.30	-1.97	14.94
1	.200e+19	6.69	1.8561	18.00	36.00	12.20	23.96	1.32	1.97	11.68
8	.700e+18	6.59	0.9067	20.00	35.00	13.10	23.34	0.64	-1.94	12.00
8	.700e+18	6.59	1.5503	20.00	35.00	13.60	23.43	1.28	-2.14	13.16
8	.700e+18	6.59	1.0220	20.00	35.00	14.65	25.77	0.94	-1.94	11.98
1.	.100e+19	6.66	3.2244	20.00	39.00	15.21	28.72	0.64	2.00	15.25
1.	.300e+19	6.71	2.3848	20.00	39.00	15.32	30.37	0.09	1.87	14.52
1	.300e+19	6.71	2.1478	20.00	39.00	15.60	29.92	-0.33	2.06	15.62
1	.060e+19	6.65	1.8713	18.48	36.50	11.18	23.34	1.37	1.97	11.66
1.	.060e+19	6.65	6.2200	17.42	35.25	12.88	23.39	1.75	-2.06	12.00
1.	.060e+19	6.65	2.1100	10.01	34.50	7.56	24.55	1.14	-1.93	11.27
andard	parameters of	of the	source. Mo	refers to se	eismic mom	ent. Mome	ont magnitu	de Mw	is comput	ted as Mw :
tandard (9.05). The Beroza (2	parameters of effective so 2000). The sl	of the urce di ip centi	source. Mo imensions (or iroid (Xc, Yc, i	refers to se Eff. Dimens Zc) is estima	eismic mom ions) is com tted as slip-v	ent. Mome nputed from weighted av	ent magnitu autocorrel eraged coo	de Mw ation wic rdinates	is compu dth of the s s (x, y, and :	ted as Mw = ilip distribution z)
tandard (9.05). The Beroza ()	parameters of effective so 2000). The si nallovic	of the urce di ip cent	source. Mo imensions (or iroid (Xc, Yc, -	refers to se Eff. Dimens Zc) is estima	eismic mom tions) is com tted as slip-v	ent. Mome aputed from weighted av	ent magnitu autocorrel eraged coo	de Mw ation wic rdinates	is compu dth of the s s (x, y, and	ted as Mw : slip distribution z)
andard (9.05). The Beroza (?	gallovic	of the urce di ip cent	source. Mo imensions (or iroid (Xc, Yc, - gallovic3	refers to se Eff. Dimens Zc) is estima hobyT	eismic mom ions) is corr tted as slip-v hobyY1	hoby	autocorrel eraged coc	de Mw ation wic rdinates	is computed the state of the st	ted as Mw = slip distribution z)
allovic 23.27	garameters of effective so 2000). The sl gallovic 18.3	of the urce di ip cent 2 g 1	source. Mo imensions (or iroid (Xc, Yc,) gallovic3 18.61	refers to se Eff. Dimens Zc) is estima hobyT 35.01	eismic mom ions) is com ted as slip-v hobyY1 37.78	hent. Mome puted from veighted av	int magnitu autocorrel eraged coc /3 ma /0 26.	de Mw ation wic rdinates ai s 83	is compu dth of the s s (x, y, and somala 31.13	ted as Mw silp distribution z) somala1 23.68
allovic 23.27 19.58	gallovic 18.3 18.3 18.3 18.3	2 g	source. Mo imensions (or roid (Xc, Yc,) gallovic3 18.61 20.17	refers to se Eff. Dimens Zc) is estima hobyT 35.01 23.59	eismic mom ions) is com ted as slip-v hobyY1 37.78 18.01	hent. Mome puted from weighted av hoby Y 25.5 15.7	int magnitu autocorrel eraged coo 73 ma 10 26. 7 2.	de Mw ation wid rdinates ai s 83 86	is compu dth of the s s (x, y, and somala 31.13 32.23	ted as Mw = slip distribution z) somala1 23.68 8.77
allovic 23.27 19.58 0.00	gallovic 18.3 5.7 11.1	2 g 1 2	source. Mo imensions (or roid (Xc, Yc,) gallovic3 18.61 20.17 9.78	hobyT 35.01 23.59 41.62	hobyY1 37.78 18.01 25.06	hoby Y 25.5 15.7 31.1	rit magnitu autocorrel eraged coc 73 ma 00 26. 77 2. 33 19.	de Mw ation wid rdinates ai s 83 86 61	is compute the of the s s (x, y, and somala 31.13 32.23 34.91	somala1 23.68 8.77 18.55
allovic 23.27 19.58 0.00 11.15	gallovic 11.1 0.000	2 g 1 2 5 0	source. Mo imensions (or roid (Xc, Yc,) gallovic3 18.61 20.17 9.78 11.05	refers to so Eff. Dimens Zc) is estima hobyT 35.01 23.59 41.62 30.57	hobyY1 37.78 18.01 25.06 21.34	hoby Y 25.5 15.7 31.1 16.9	rit magnitu autocorrel eraged coc 73 26. 77 2. 3 19. 0 7.	de Mw ation wic rdinates ai s 83 86 61 56	is compute th of the s s (x, y, and somala 31.13 32.23 34.91 25.24	somala1 23.68 8.77 18.55 9.81

10.00	0.12	20.11	20.00	10.01	10.11	2.00	02.20	0.11	
0.00	11.15	9.78	41.62	25.06	31.13	19.61	34.91	18.55	
11.15	0.00	11.05	30.57	21.34	16.90	7.56	25.24	9.81	
9.78	11.05	0.00	45.72	18.82	26.96	23.76	31.04	16.40	
41.62	30.57	45.72	0.00	36.45	30.62	23.54	52.43	26.71	
25.06	21.34	18.82	36.45	0.00	30.57	22.70	46.39	18.90	
31.13	16.90	26.96	30.62	30.57	0.00	19.77	40.63	18.52	
19.61	7.56	23.76	23.54	22.70	19.77	0.00	29.43	11.55	
34.91	25.24	31.04	52.43	46.39	40.63	29.43	0.00	24.75	
18.55	9.81	16.40	26.71	18.90	18.52	11.55	24.75	0.00	
e of dis	of dissimilarity between pairs of models obtained using normalized square metric								

- Generate an *m*-dimensional configuration in Euclidian space based on (dis-)similarity between pairs of 2D random fields (e.g. slip models) Visualize these point-configurations in a lower-dimensional (2D, 3D) representation
- Method:
- Construct matrix **D** with elements that measure dissimilarity (e.g. SE, AE) Construct matrix **B** from **D**, by double-centering **D** (for symmetry purposes) Apply SVD to **B**, such that $\mathbf{B} = \mathbf{V} \mathbf{A} \mathbf{V}^{\mathsf{T}}$
- Select *n*-points in *p*-dimensional space from $x_{ii} = V_{ii} \lambda_i^{\frac{1}{2}}$, i = 1 ... n, j = 1 ... pCoordinates of x are constructed such that either a mean-model is the ference, located then in the center of the point cloud, or that any selected model (known solution) becomes the reference



Inv3 – Large strike-slip rupture in Southern California

- Kinematic rupture, with heterogeneity in source parameters
- Near-field synthetics in 3D Earth structure (Vs_{min} 500 m/s)



Inv4 – Complex-geometry blind-thrust scenario in Southern California

- Currently under development, to be released Summer 2015
- 2. Multiple synthetic datasets, computed in 3D Earth structure
- Meta-data of rupture and structure are not revealed to modelers

PRELIMINARY CONCLUSIONS & OUTLOOK

- Through a series of benchmarks we aim at being able to discriminate "strong" sourceinversion methods from "weak" ones, and to identify where deficiencies could be • The project & efforts are ongoing, but already have been used to develop and test new methods, or to 'calibrate' existing ones
- The Spatial Prediction Comparison Test (SPCT) seems to be a useful tool to quantify

**** SIV Participants ****

Jean-Paul Ampuero, Morgan Page, Danijel Schorlemmer; Kimiyuki Asano, Matthieu Causse, Wenyuan Fen, Frantisek Gallovic, Hoby Razafindrakoto, Surendra Somala, Cedric Twardzik, Rongjiang Wang. Special thanks to: Walter Imperatori, Kiran Thingbaijam, Olaf Zielke, Ling Zhang, Martin van Driel, Yuji Yagi, Ryo Okuwaki Fred Pollitz, and many others who contributed to the SIV efforts one way or the other.