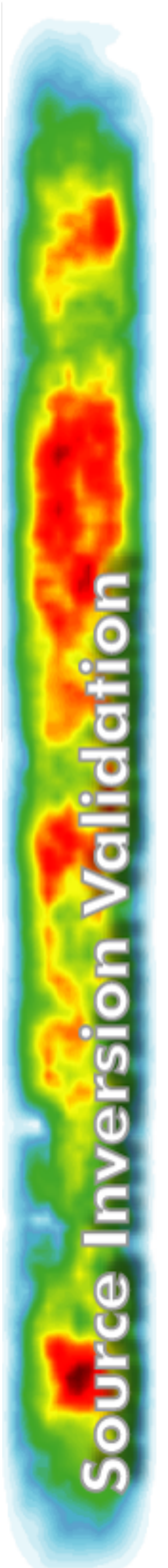


Uncertainty Quantification and Quality Appraisal for Finite-Fault Earthquake Source Inversions

P. Martin Mai & many, many SIV-participants*   martin.mai@kaust.edu.sa   http://equake-rc.info/siv

an NSF+USGS center

SUMMARY

Finite-fault source inversions estimate kinematic rupture parameters of earthquakes using a variety of available data sets and inversion approaches. Rupture models are obtained by solving an inherently ill-posed inverse problem, subject to numerous *a priori* assumptions and noisy observations. Despite these limitations, near real-time source inversions are becoming increasingly popular, while we still face the dilemma that uncertainties in source inversions are essentially unknown. Yet, the accurate estimation of earthquake rupture properties, including proper uncertainty quantification, is critical for earthquake seismology and seismic hazard analysis, as they help to characterize earthquake complexity across all scales.

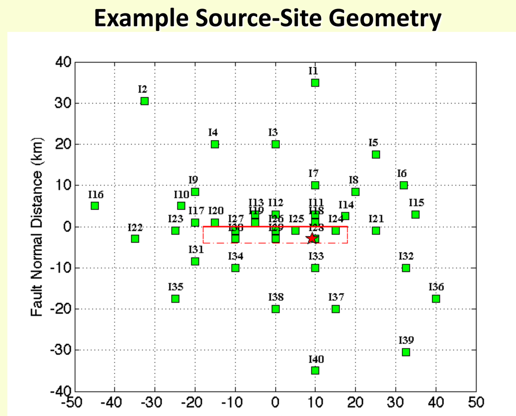
The "Source Inversion Validation" (SIV) project is a collaborative international multi-institutional effort to examine current state-of-the-art in earthquake source inversion, and to develop and test novel source inversion approaches. Through a series of benchmark exercises of varying degree of complexity, we test inversion methods, and evaluate their performance through different comparative metrics. We quantify the intra-event variability in rupture models, which is evident for past earthquakes in the SRCMOD database (http://equake-rc.info/srcmod), and propose metrics to rank earthquake rupture models.

Here, we summarize the SIV-efforts and latest results, and describe several quantitative metrics to parameterize the similarity (or dissimilarity) of kinematic source properties (e.g. slip on the fault) that help to assess the quality and model robustness of finite-fault source models.

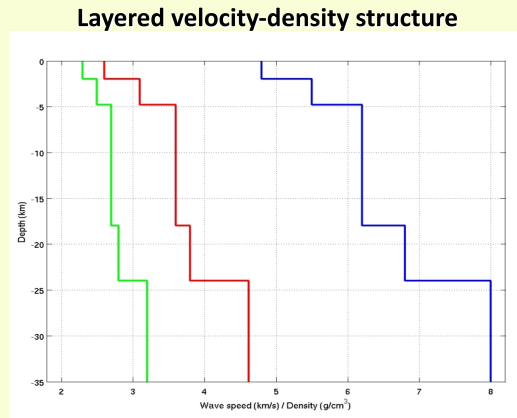
APPROACH

- Series of benchmarks with varying degree of complexity, with and without "noise" in the data and various levels of information on meta data (e.g. fault geometry, velocity-density structure)
- All benchmarks remain accessible for all interested users; only for the most recent test the solution (input model) is not released
- Use various statistical metrics to quantitatively compare and "rank" models
- For the initial Green's function testing and the first benchmarks, Earth structure and the source-station geometry remains fixed to simplify the work

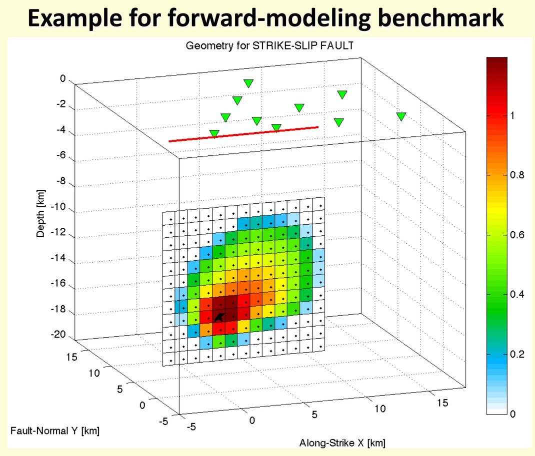
Example Source-Site Geometry



Layered velocity-density structure



Example for forward-modeling benchmark



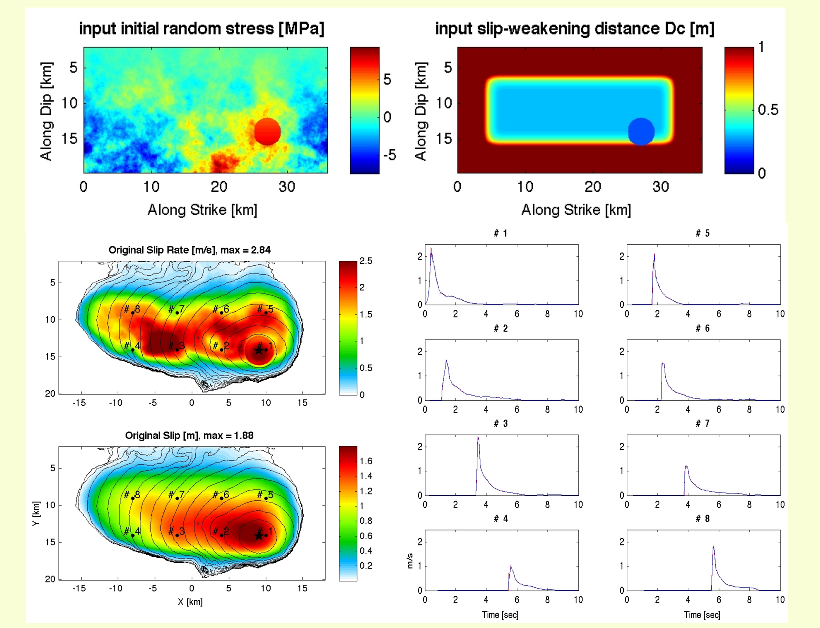
Sequence of Benchmark Exercises

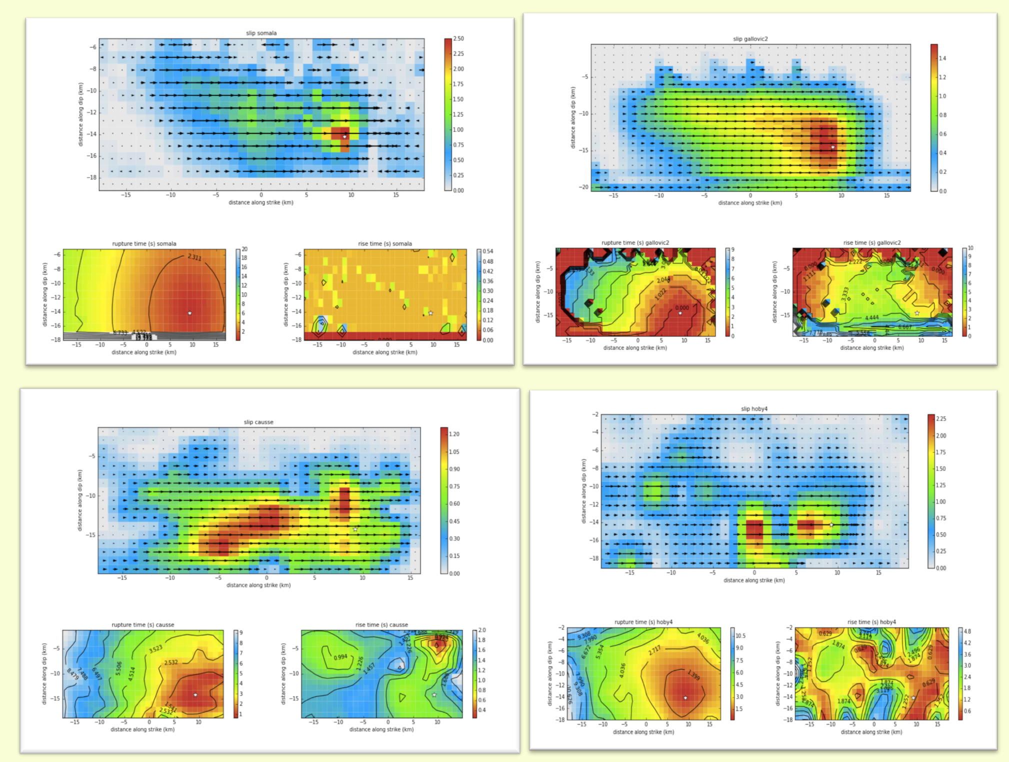
1. Point-source forward-calculations for Green's function testing
2. Forward-modeling cases for two "simple" extended-fault kinematic cases
3. Inv1: Inversion for "simple" M 6.5 strike-slip dynamic rupture model
4. Inv2: Inversion for kinematic M 7 normal-faulting scenario, incl. uncertainties in the Green's functions (through 3D scattering)
5. Inv3: teleseismic case for very large strike-slip rupture in Southern California
6. Inv4: *Complex-geometry blind-thrust earthquake in Southern California*

INITIAL RESULTS & STATISTICAL METRICS

INV 1: M 6.5 strike-slip scenario

- Input for spontaneous dynamic rupture model using heterogeneous initial stress, to generate rupture model and near-fault synthetics for source-site geometry and velocity-density structure (shown on previous panel)
- Kinematic source properties of slip-rate (top left) and final slip (bottom left), and resulting source-time functions at selected points on the rupture plane (bottom right)
- Despite noise-free synthetic data and perfectly known meta-data (Earth structure; geometry), the inversion solution show substantial variations (below: four example solutions from a set of 11 submitted solutions)

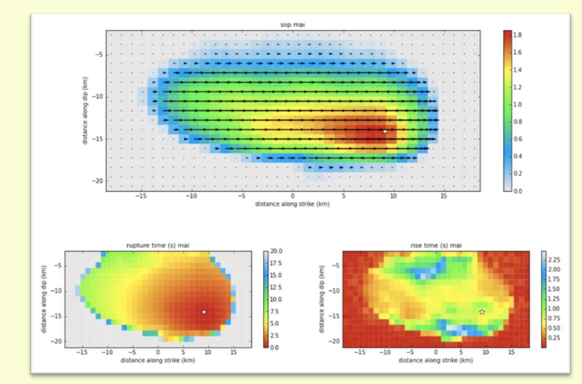




Slip-inversion results for INV1 benchmark.

Top panel shows slip on the fault; bottom left: rupture time; bottom right: rise time

Below: the reference solution

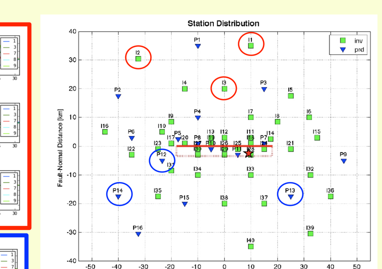


Statistical Metrics to Quantify Source-Model Differences

1. Waveform misfits are computed using standard norms (L1, L2, variance reduction, cross-correlation)
2. We also report time-frequency envelope (TFE) misfits and time-frequency phase (TFP) misfit to determine goodness-of-fit criteria (Kristekova et al., 2006, 2009)
3. Differences in the slip distributions are assessed using the **Spatial Prediction Comparison Test** (SPCT; Hering & Genton, 2011; Zhang et al. 2015) and **multi-dimensional scaling** (Razafindrakoto et al. 2015) to rank the models
4. The statistical tools are developed, tested, and calibrated on a range of synthetic cases, and can be used in case a known solution exist.

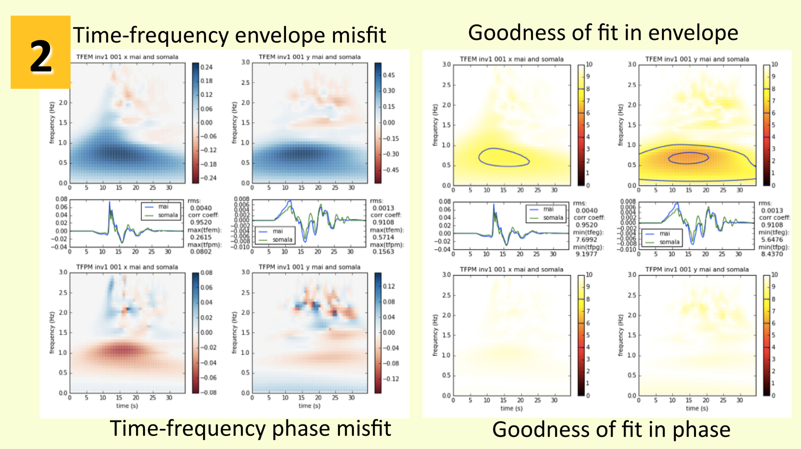
→ These statistical tools can be applied also to slip models of past earthquakes (with unknown true solution) by defining an appropriate test or "mean" distribution

1



Example waveforms misfits at sites for the inversion (red) and "blind-predicted" sites (blue; not used in the inversion

2



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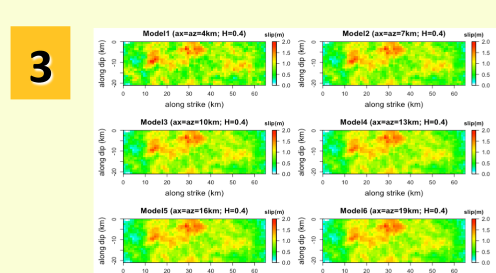
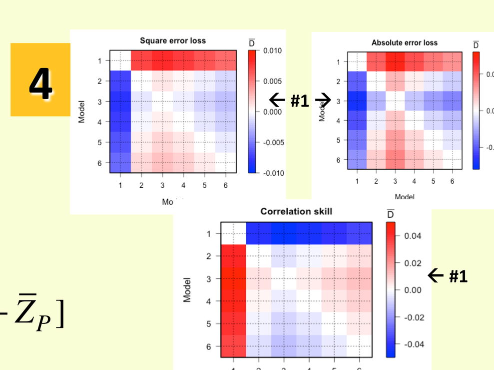


Synthetic test models, with identical random seed but different correlation lengths

4



Top (SE, AE): Negative values (blue) indicate that the case named in the corresponding row is the better model
Bottom (CS): values (red) indicate that the case named in the corresponding row is better based

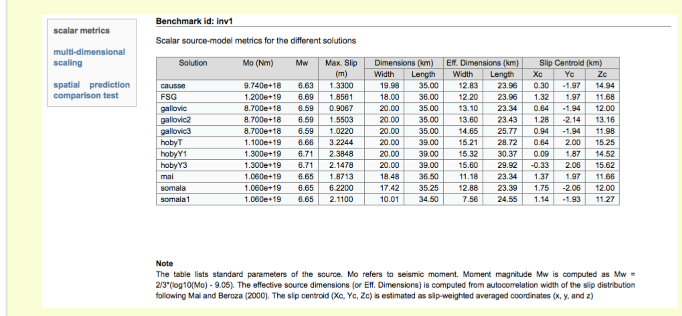
Brief summary of notation

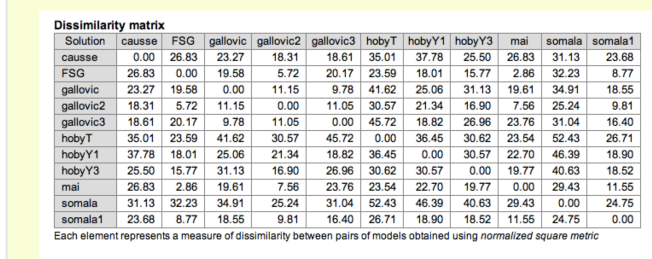
- Spatial process Z(**s**) at locations **s** $\{Z(\mathbf{s}) \in \mathbb{R} : \mathbf{s} \in D \subset \mathbb{R}^2\}$
- General loss function for prediction and realization $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)]$
- Squared-error loss (SE) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = [Z(\mathbf{s}_i) - \hat{Z}_P(\mathbf{s}_i)]^2$
- Absolute-error loss (AE) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = \left|Z(\mathbf{s}_i) - \hat{Z}_P(\mathbf{s}_i)\right|$
- Correlation skill (CS) $g[Z(\mathbf{s}_i), \hat{Z}_P(\mathbf{s}_i)] = \frac{n}{(n-1)\hat{\sigma}_Z \hat{\sigma}_P}[Z(\mathbf{s}_i) - \bar{Z}][\hat{Z}_P(\mathbf{s}_i) - \bar{Z}_P]$

CURRENT & FORTHCOMING BENCHMARKS

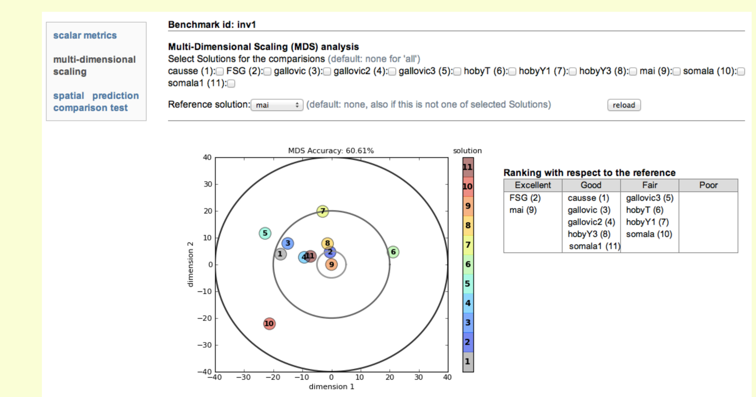
Statistics for INV 1

- SPCT and multi-dimensional scaling show that for the INV 1 case, model solution #2 is best

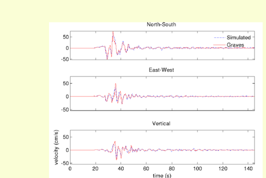


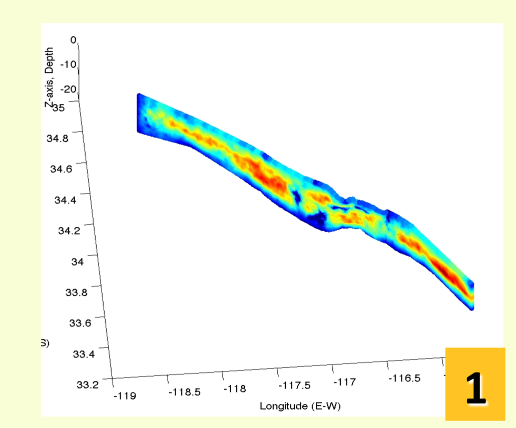


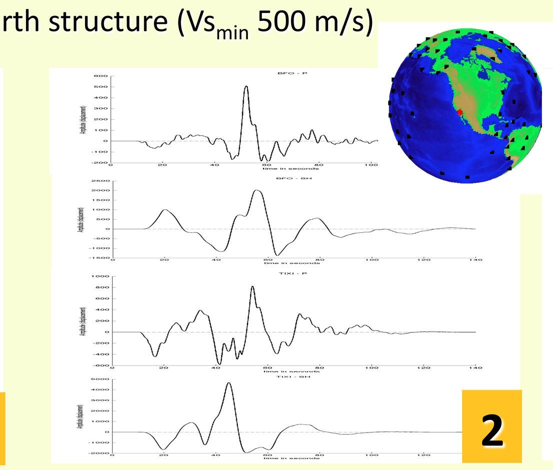
- **Multi-dimensional scaling**
  - Generate an *m*-dimensional configuration in Euclidian space based on (dis-)similarity between pairs of 2D random fields (e.g. slip models)
  - Visualize these point-configurations in a lower-dimensional (2D, 3D) representation
- **Method:**
  - Construct matrix ***D*** with elements that measure dissimilarity (e.g. SE, AE)
  - Construct matrix ***B*** from ***D***, by double-centering ***D*** (for symmetry purposes)
  - Apply SVD to ***B***, such that $\mathbf{B} = \mathbf{V}\Lambda\mathbf{V}^T$
  - Select *n*-points in *p*-dimensional space from $x_{ij} = V_{ij}\lambda_j^{1/2}$, $i = 1 .. n$, $j = 1 ... p$
  - Coordinates of *x* are constructed such that either a mean-model is the reference, located then in the center of the point cloud, or that any selected model (known solution) becomes the reference

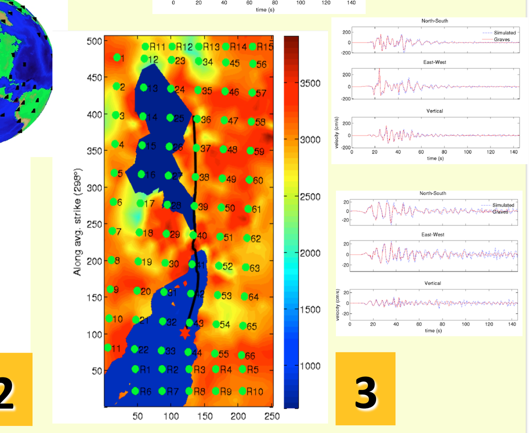


- **Inv3 – Large strike-slip rupture in Southern California**
  1. Kinematic rupture, with heterogeneity in source parameters and somewhat complicated geometry
  2. Teleseismic synthetics (P, SH)
  3. Near-field synthetics in 3D Earth structure (Vs_{min} 500 m/s)









- **Inv4 – Complex-geometry blind-thrust scenario in Southern California**
  1. Currently under development, to be released Summer 2015
  2. Multiple synthetic datasets, computed in 3D Earth structure
  3. Meta-data of rupture and structure are not revealed to modelers

PRELIMINARY CONCLUSIONS & OUTLOOK

- Through a series of benchmarks we aim at being able to discriminate "strong" source-inversion methods from "weak" ones, and to identify where deficiencies could be
- The project & efforts are ongoing, but already have been used to develop and test new methods, or to 'calibrate' existing ones
- The Spatial Prediction Comparison Test (SPCT) seems to be a useful tool to quantify how well a given 2D field (slip model) "fits" a reference solution
- Using a multidimensional scaling approach allows to further quantify in which sense the models are different (amplitude; patch location ..), and to propose some form of ranking for the models)

** SIV Participants **

Jean-Paul Ampuero, Morgan Page, Danijel Schorlemmer; Kimiyuki Asano, Matthieu Causse, Wenyuan Fen,Frantisek Gallovic, Hoby Razafindrakoto, Surendra Somala, Cedric Twardzik, Rongjiang Wang. Special thanks to: Walter Imperatori, Kiran Thingbaijam, Olaf Zielke, Ling Zhang, Martin van Driel, Yuji Yagi, Ryo Okuwaki Fred Pollitz, and many others who contributed to the SIV efforts one way or the other.